AN INTRODUCTORY LOGIC

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NEW EDITION, REVISED AND CORRECTED

New York
THE MACMILLAN COMPANY
LONDON: MACMILLAN & CO., LTD.
1902

Set up and electrotyped September, 1898. Reprinted July, 1899. New Edition, Revised and Corrected, March, 1900 ; April, October, 1901 ; July, 1902.

Norwood Press
J. S. Cushing & Co. — Berwick & Smith
Norwood Mass. U.S.A.

PREFACE

THIS volume is intended primarily as a text-book for college students, and grew out of my lectures on Logic to undergraduate classes in Cornell University. It aims at being both practical and theoretical. In spite of the obvious deficiencies of formal Logic as a theory of the nature of thought, I am convinced that it is one of the most valuable instruments in modern education for promoting clear thinking, and for developing critical habits of mind. J. S. Mill, speaking in the *Autobiography* of the discipline which he received from working logical exercises, expresses the following opinion: "I am persuaded that nothing, in modern education, tends so much, when properly used, to form exact thinkers, who attach a precise meaning to words and propositions, and are not imposed on by vague, loose, or ambiguous terms." Although in treating the syllogistic Logic I have followed to a large extent the ordinary mode of presentation, I have both here, and when dealing with the Inductive Methods, endeavoured to interpret the traditional doctrines in a philosophical way, and to prepare for the theoretical discussions of the third part of the book.

The advisability of attempting to include a theory of thought, or philosophy of knowledge, even in outline,

in an elementary course in Logic, may at first sight appear doubtful. It seems to me, however, that this inclusion is not only justifiable, but even necessary at the present time. Psychology is no longer a 'philosophy of mind'; but, under the influence of experimental methods, has differentiated itself almost entirely from philosophy, and become a 'natural' science. As a natural science, it is interested in the structure of the mental life, — the characteristics of the elementary processes, and the laws of their combination, — and not primarily in the function which ideas play in giving us knowledge. It is clear that psychology does not undertake to describe all that mind is and does. It belongs to Logic to investigate intelligence as a knowing function, just as it is the task of Ethics to deal with the practical or active mental functions.

The practical question still remains as to whether this side of Logic can be made profitable to students who have had no previous philosophical training. I am well aware of the difficulty of the subject, but my own experience leads me to believe that the main conceptions of modern logical theory can be rendered intelligible even to elementary classes. Of the incompleteness and shortcomings of my treatment I am quite conscious; but I have endeavoured to make the matter as simple and concrete as possible, and to illustrate it by means of familiar facts of experience.

For a number of the practical questions and exercises, I am indebted to Professor Margaret Washburn of Wells College; others are original, or have been collected in the course of my reading. I have also

taken a number of arguments from the examination papers of different universities, and from various works on Logic, especially from Jevons's *Studies in Deductive Logic*, from the little volume entitled *Questions on Logic* by Holman and Irvine (2d ed., London, 1897), and from Hibben's *Inductive Logic*.

In writing the book, I have been under obligation to a large number of writers and books. My heaviest debt is doubtless to Bosanquet, and perhaps next in order I am under obligations to Mill, Jevons, Sigwart, and Bradley. I have also derived help from Minto's *Logic, Deductive and Inductive*, the chapter on 'Reasoning' in James's *Principles of Psychology*, J. H. Hyslop's *Elements of Logic*, and from other works to which reference is made throughout the book.

My colleagues in the Sage School of Philosophy have kindly aided me from time to time with advice and encouragement, and I have also received valuable suggestions from other teachers of Logic with whom I have talked and corresponded. In particular, I wish to express my obligations to my former colleague, Professor James Seth, who read nearly all of the book in manuscript, and to Dr. Albert Lefevre, who kindly assisted me in reading the proofs.

J. E. C.

CORNELL UNIVERSITY,
August, 1898.

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AN INTRODUCTORY LOGIC

INTRODUCTION

CHAPTER I

THE STANDPOINT AND PROBLEM OF LOGIC

§ 1. **Definition of the Subject.** — Logic may be defined as the science of thought, or as the science which investigates the process of thinking. Every one knows, in a general way at least, what is meant by thinking, and has noticed more or less consciously some of its peculiarities. Thinking is the intellectual act by means of which knowledge is obtained. We do not really *know* any fact until we *think* it; that is, until the mind sets it in its proper relation to the other parts of its experience, and thus comes to understand its true meaning. We make a distinction, for example, between what has come to us through report or hearsay, and conclusions which we have reached by our own thinking. 'I have *heard*,' we say, 'that A is dishonest, but I do not *know* it.' That is, this fact has not been reached as a result of our own thinking, and cannot therefore claim the title of knowledge. On the other hand, that the earth is round, is not a mere matter of hearsay for an educated man. It is a piece of *knowledge*, because it is a conclusion which he has reached by thinking, or by putting together various facts for himself.

Logic, then, in dealing with thinking, is concerned with the process by which knowledge is obtained. In defining it as a science, we mean that it seeks to substitute exact and systematic knowledge regarding the nature of thought for the popular notions to be found in everyday life. Like all the sciences, logic has to correct and supplement ordinary knowledge. It is its mission to help us to understand more exactly and completely the way in which thinking goes on, and to discover the laws which are followed in gaining knowledge.

But it is also the business of a science to systematize facts. Logic, then, cannot content itself with a mere description of this or that kind of thinking, in isolation from other ways in which we think. It must also deal with the way in which the various kinds of thinking are related. For example, we apply such terms as 'conception,' 'judgment,' 'induction,' and 'deduction' to different intellectual operations, and give the distinguishing characteristic in each case. But it is necessary as well to understand how these processes are related. Since all thinking has one end, the discovery of truth, the various intellectual operations must mutually coöperate and assist in this result. All of the logical processes, then, stand in relation to one another. They are all parts of the one intelligence, though they may well represent different stages or steps in its work of obtaining knowledge. It becomes the business of logic, then, to show us the organic structure of thought. In other words, it must furnish a comprehensive view of the way in which intelligence

acts, and the part which processes like 'conception,' 'judgment,' 'induction,' etc., play.

(1) The word 'logic' is derived from the adjective corresponding to the Greek noun λόγος, which signifies either a complete thought, or a word as the expression of that thought. The singular form of the adjective λογική, from which the English word is derived, was supposed to qualify either ἐπιστήμη as applying to the theoretical *science* of logic, or τέχνη as referring to the practical application of its rules and as affording guidance in the *art* of correct reasoning. We shall have to raise the question in a subsequent section how far it is possible to regard logic as an art, or a system of rules which teach us how to reason correctly.

(2) We have defined logic as the science of the operations and processes of thought, or as the science of thinking. It is evident, however, that this definition does not carry us very far unless we know what thinking means. And to gain a clearer idea of this common term may be said to be the problem of logic. This is, however, by no means as easy a task as may at first appear. Familiar words and phrases often conceal difficulties. They are constantly repeated without reflection, and this very frequency of repetition is likely to prevent us from trying to gain any clear ideas regarding the nature of the objects which they denote. It is only when we become discontented with our knowledge regarding any subject, when doubts arise whether we really understand the meaning of the words which we use, that we attempt to make our knowledge scientific, *i.e.*, to gain clear, definite, and systematic ideas. This can perhaps be made clearer by considering the main differences between an educated and an uneducated man. The educated man has, of course, a great deal more information than the other, and his knowledge is more definite and systematic. But a second and more important distinction is found in the attitude of mind which education begets. The educated man is desirous of knowing more, because he is sensible of his own ignorance. The uneducated man, on the other hand, supposes that he knows all about things whose names are familiar to him. He can settle puzzling theological or political problems off-hand in a way which is per-

fectly satisfactory to himself, without study, and almost without reflection.

It is clear that no intellectual salvation is possible for a man so long as he remains in this state of mind. A sense of one's own ignorance is the beginning of wisdom. Socrates, one of the great pioneers of science among the Greeks, and especially of the sciences of logic and ethics, was so firmly convinced of this that he made it the business of his life to go about the streets of Athens and convince those "who thought they were wise and were not wise," of their ignorance. "And because I did this," he says naïvely, "many of them were angry, and became my enemies."

§ 2. **Relation to Psychology.** — It may aid us in obtaining a clearer view of what thinking is, if we compare the general standpoint of logic with that of psychology. Both of these sciences deal with what goes on in mind or consciousness, and are thus opposed to the so-called objective sciences, which are all concerned with some group or field of external facts. But in spite of this agreement, there is an important distinction between logic and psychology. In the first place, psychology deals with all that there is in mind. It describes pleasures and pains, acts of will, and the association of ideas, as well as what is usually called logical thinking. But logic does not differ from psychology simply by being less inclusive than the latter. It is true that, from the standpoint of psychology, the thought-process is merely a part of the mental content, which has to be analyzed and described like anything else which goes on in consciousness. Thinking has doubtless for psychology peculiar marks or characteristics which distinguish it from other related processes like those of association; but when these have

been found, and the psychological description of thinking is complete, the question with which logic deals has not yet been raised. For logic, as we shall see presently, adopts a different standpoint, and investigates with a different end in view.

The important difference is this: In psychology we are interested in the content of consciousness *for its own sake,* and just as it ~~stands. We try to find~~ out what actually goes on in our minds; and to describe it just as we should any event which occurs in the external world. But in logic the question is not: What are mental processes? but rather: What knowledge do they give us, and is this knowledge true or false? Logic, in other words, does not regard the way in which ideas exist, and is not interested in them *for what they are*, but rather in the purpose which they subserve in affording us knowledge of something beyond themselves. Psychology, in its description of conscious states, inquires regarding their quality, intensity, duration, etc., and the ways in which they combine with each other to form complex ideas. The problem with which logic is concerned, on the other hand, has reference to the *value* of ideas when they are taken to represent facts in the real world. In other words, the question which logic raises is not regarding the *actual character* of ideas as existing processes, but regarding their value or significance as pieces of knowledge.

(1) The relation between logic and psychology may perhaps be illustrated by referring to that which exists between morphology and physiology. Morphology deals with the form and structure of living organisms, and physiology with the various acts and func-

tions which these organisms discharge. Thus we speak of the former as the science of form or structure, and of the latter as the science of function. In the same way, psychology may be said to deal with the actual structure of mental processes, and logic with the part which they play in giving us knowledge.

It must be noticed, however, that this is a distinction made for purposes of investigation, and does not denote that structure and function have nothing to do with each other. On the contrary, some knowledge of the function is often necessary in order to understand the structure of an organ; and, on the other hand, it is usually true that the nature of a function only becomes completely intelligible when the character of the mechanism with which it works is known. And the same holds true, I think, of the relations between psychology and logic. Although it has been found profitable when dealing with consciousness, as in the biological realm, to investigate the nature of structure and function separately, yet here, as there, the two lines of inquiry cross each other; for it is beyond question that the knowledge we obtain by thinking is largely dependent upon the character (quality, intensity, etc.) of the actual processes in consciousness. To understand the nature of a logical idea, then, it is often necessary to refer to the psychological facts and their actual mode of behaviour. And it is equally true that one cannot carry on a psychological investigation into the nature of mental processes without taking account, to some extent, of the part which they play in giving us knowledge. No psychology is able to take ideas simply as existing conscious processes to which no further meaning or importance attaches; it is only with reference to the function they perform as *knowing* states that their own peculiar character can be understood. In other words, the intellectual activities and purposes of mind must be presupposed in psychology, though this science, for the most part, goes its way as if the ideas were not cognitive at all. At least this seems to be true of the 'new' or experimental psychology as opposed to the philosophies of mind.

(2) It would of course be presumptuous, as well as utterly useless, for any writer to draw a hard and fast line between logic and psychology, and to forbid others to overstep it. In attempting to dis-

cover the dividing line between two closely related sciences one must be guided by the procedure of those who are working in the fields which it is proposed to divide. Now, it must be admitted that by no means all of the recent writers in psychology limit the sphere of their science in the way above described; that is, there are certain psychologists who do not confine their attention to the mere mental processes as such, but include in their investigations the further problem regarding the part which these processes play in giving us knowledge. Thus in Professor James's *Principles of Psychology* there is an excellent chapter on 'Reasoning' which certainly contains as much logical as psychological matter. In the same way, one finds problems of knowledge discussed in the psychological writings of Professor Ladd, and also, to some extent, in the recent work by Mr. Stout entitled *Analytic Psychology*. In spite of this, it is evident that the tendency of the 'new,' or laboratory psychology, is towards a sharper differentiation of its problems from those of logic. The 'natural science of psychology' is interested in the conscious process as an event in time with certain definitely ascertainable characteristics. It is perhaps not a matter of great moment whether the name 'psychology' be limited to this kind of inquiry, or whether philosophical inquiries regarding the nature of knowledge be also included under it. I have assumed, however, in this section, that psychology is now being differentiated from the more general inquiries regarding the nature of mind, and that it has taken for its field of investigation the nature of mental processes regarded merely as mental processes.

Consider a little further the nature of the ideas with which logic deals. Every idea, as we have seen, not only exists in some definite fashion in some particular consciousness, connected with certain other ideas, and with a definite quality, intensity, etc., but it has a meaning or significance as a piece of knowledge. It not only *is* something, but it also stands for or *signifies* something. Now it is not with the existence, but with

the meaning side of ideas that logic has to do. A logical idea, or piece of knowledge, is not merely a modification of consciousness which exists in the mind of some individual at a particular time. For example, the proposition: 'The three angles of a triangle are equal to two right angles,' will give rise to a number of definite psychological processes (probably auditory or visual in character) in the mind of any individual. These processes would also probably differ in character in the case of two persons. The *meaning* of the proposition, however, is distinct from the definite processes which arise in particular minds. The proposition has a significance as an objective fact, or piece of knowledge, outside my mind; the psychological images or processes may differ for different persons, but the fact expressed is the same for all minds and at all times.

§ 3. **Logic as a Science and an Art.** — We have defined logic as the science of thought, but it has often been pointed out that there are equally strong reasons for considering it to be an art. Jevons makes the distinction between a science and an art very clear by saying that "a science teaches us to know, and an art to do." A science is interested in the discovery of facts and laws without any thought of what use may be made of this knowledge; an art, on the contrary, gives practical guidance and direction for some course of action. The question before us, then, is this: Does logic merely give us knowledge about the ways in which we think, or does it also help us to think rightly?

Before we attempt to answer this question, we must

note that practical rules of action are based upon scientific knowledge. An art, in other words, depends upon science, and grows in perfection with the advance of scientific knowledge. Thus medicine, as the art of healing, is founded upon the sciences of chemistry, physiology, and anatomy, and it is because of the great discoveries which have been made in these fields within recent years, that it has been able to advance with such gigantic strides. Again, the art of singing, in so far as it is an art which can be taught and learned, depends upon a knowledge of the physical and physiological laws of the vocal organs. An art, then, always presupposes a certain amount of science, or knowledge, and is simply the application of this knowledge to some practical purpose. In some cases the application is very obvious and direct; in others, it is much more difficult to determine; but, in general, there is always this relation between theory and practice, between knowing and doing.

From what has been already said, it will be evident that logic must first be a science before it can become an art. Its first business must be to investigate the nature of thought, and to attempt to discover the different forms which the latter assumes in the course of its development. So that we were right in defining it as primarily a science. But the further question remains: How far is it possible to apply the laws of logic after they have been discovered in such a way as to obtain directions how to reason correctly in every case? Can we not apply our knowledge of the laws of thought in such a way as to get a complete art of reasoning, just as

the laws of chemistry and biology are applied in medicine?

It is no doubt true in logic, as everywhere, that scientific knowledge is capable of practical application. But I do not think that logic can be regarded as an art, in the sense that it furnishes a definite set of rules for thinking correctly. There is an important distinction in this case which must not be left out of account. The physical, and even the biological sciences, deal with things whose way of acting is perfectly definite and uniform. The character of any of the physiological functions, as, *e.g.*, digestion, may be comparatively complex and difficult to determine, but it always attains its end through the use of the same means. When once its laws are understood, it is not difficult to prescribe just how the proper means may always be secured for the attainment of the desired end. But thinking has much more flexibility in its way of acting. We cannot say with the same definiteness as in the cases we have been considering, that in order to reach a certain end we must use a definite set of means. It is not possible, that is, to say: If you would learn what is true about this subject, you must follow this rule and that in your thinking. Logic, it seems to me, cannot be regarded as an art like photography, or even like medicine; for it is not possible to lay down definite rules for the guidance of thinking in every case. What we can do, is to show the method by which new truths have been discovered, and the general conditions which must always be fulfilled in reasoning correctly. And it is also possible to point out the more common errors which arise when these

conditions are violated. But it is beyond the power of logic to formulate any definite set of rules for the guidance of thinking in every case.

We have found that we must give up all extravagant hopes of the practical advantages to be gained from a study of logic. There is no set of rules which will make us infallible reasoners. That being admitted, the question may be raised as to the utility of the study. What will it profit us to devote ourselves to this subject? It might be a sufficient answer to point out that this question presupposes that knowledge has always some ulterior motive. The assumption upon which it is based is, in other words, that the practical advantages arising from any study furnish the only justification for undertaking it. But it is scarcely necessary to say that this is not an attitude which any student should adopt. A student is one who prosecutes a study for its own sake, with no other motive than the desire to know. And to such a person logic should not be without interest. For as we have seen, it is an inquiry into the nature of intelligence. Its results, therefore, are not in themselves less interesting or less important than a knowledge of the various forms of geological formation, or of plant or animal life. "If it is regarded as a valuable achievement," says Hegel, "to have discovered sixty odd species of parrot, a hundred and thirty-seven species of veronica, and so forth, it should surely be held a far more valuable achievement to discover the forms of reason."[1]

The necessity of devoting oneself to a science quite unselfishly cannot be too strongly enjoined, nor the evils which arise when one begins a study greedy 'for quick returns of profit,' too often emphasized. Nevertheless, since the question has been raised, it would not be just to refuse altogether to speak of the particular results

[1] Hegel, *Werke*, Bd. V., p. 139. Quoted by Bosanquet at the beginning of his work on Logic.

arising from a study of logic. As we have seen, we cannot hope to become infallible reasoners by its aid. It is just as true here as in any other field, however, that knowledge is power, and ignorance synonymous with weakness. For even if one resolves never to look inside a logic book, one must nevertheless have some theory, or act upon some principle — it may be quite unconsciously — in deciding what is true and what is false. For instance, a man may act upon the principles that those things are likely to be true which are favourable to his own interests, or which agree with his own prejudices, or with the articles of his church or political party. Or again, he may regard his senses as the standards of truth. Mr. Bradley says that *if* dogs reason, they proceed upon the principle, 'what smells, exists, and what does not smell does not exist.' It is not uncommon to hear it announced: What can be perceived through the senses is true; what cannot be sensed, or is contrary to the testimony of the senses, is an absurdity. This was the standard of truth adopted, for example, by those who attempted to overthrow the Copernican theory by declaring it to be in plain contradiction to the testimony of the senses.

It seems evident, therefore, that intellectual beings cannot escape some kind of logical theory, whether they hold it consciously or unconsciously. It is clear, too, that the character of this theory will determine to a great extent their thoughts and opinions. The only question which remains is whether it is better to leave this matter entirely to chance, or to attempt to gain some clear ideas regarding the nature of thinking,

and the conditions under which knowledge arises. It can scarcely be doubted that, even from a practical point of view, a true theory is better than a false one. A man who has reflected upon the nature of proof, and the principles of reasoning, is much less likely to be deceived than one who is guided unconsciously by assumptions which he has never examined. It is always an advantage to know exactly the nature of the result at which we are aiming, and to be perfectly clear as to our own purposes. And this is just what a study of logic aids us in attaining. It helps us to understand the structure of knowledge and conditions of proof. Moreover, it engenders the habit of criticising propositions, and examining the evidence upon which they rest. Further, the importance of this study for a theory of education may well be emphasized. For education, at least so far as it undertakes to train the knowing powers of the individual, must be based upon a knowledge of the necessary laws of intelligence, and of the steps or stages which it passes through in its process of development.

§ 4. **The Material of Logic.** — The business of logic, as we have seen, is to discover the laws of thought and to show the differences which exist between real and imaginary knowledge. Where now shall we find the materials for this study? Where are the facts which are to be taken as a starting-point? It is, of course, impossible to learn directly from one's own consciousness all that thinking is, or everything of which it is capable. For, quite apart from the difficulty of observing the process of thought while it is actually going on,

no one can suppose that his own mind furnishes an example of all that thinking has done, or can do. It is necessary to take a broader view, and learn how other men think. Of course, we cannot look into the consciousness of other men, but we can study the products and results of their thoughts. The history of the way in which truth has been discovered is of the greatest importance for logic. It must not be forgotten that thought is not a thing which can be described once for all. It is rather a living activity, which is constantly showing what it is in what it does. The history of the various sciences furnishes a record of the steps by means of which thought has built up knowledge. And, in this record, we have also a revelation of the nature of the thinking process itself, and of the stages through which it has passed in the course of its development.

It is by a reflection, then, upon the nature of propositions which are universally regarded as true that the laws of logic are obtained. There is always a permanent body of knowledge which no one thinks of calling in question. Both in everyday knowledge, and in the sciences, there is always found a great number of propositions which appear true to everybody. And it is here that logic finds its material. Taking the facts and propositions which are recognized as certain by everybody, logic examines their structure in order to learn about the nature of the intellectual processes by which they have been discovered. What principles, it asks, are involved in those pieces of knowledge, and what particular acts of thought were necessary to discover them? It is only by examining various pieces of knowledge

in this way, and attempting to trace out the conditions of their discovery, that one can learn anything new regarding the laws and character of thought. In other words, there is no way of learning about thinking except by studying what it has done. The best way of getting information about what thought can do, is to study what it has already accomplished.

Every piece of knowledge, as the product of thinking, is to some extent a revelation of the nature of intelligence. But scientific knowledge — by this I mean the results of the philosophical and historical sciences as well as of the so-called natural sciences — exhibits perhaps most clearly the nature of thought. For the history of these sciences enables us to see the process of knowledge, as it were, in the making. In tracing the history of philosophical and scientific ideas, we are at the same time following the laws of the development of thought. It is this fact which makes the history of philosophy and of the various sciences so instructive. It was with this object in view, to take but a single example, that Whewell wrote his famous *History of the Inductive Sciences.* He was interested, that is, not so much in the mere facts and names with which he dealt, as in showing the nature of thinking and the methods which had been employed in gaining a knowledge of the world. This is made very clear in the introduction to another work of Whewell from which I quote: "We may best hope to understand the nature and conditions of real knowledge by studying the nature and conditions of the most certain knowledge which we possess; and we are most likely to learn the best methods of discovering truth by examining how truths, now universally recognized, have really been discovered. Now there do exist among us doctrines of solid and acknowledged merit certainly, and truths of which the discovery has been received with universal applause. These constitute what we commonly term *sciences;* and of these bodies of exact and enduring knowledge we have within our reach so large a collection that we may hope to examine them and the history of

their formation with a good prospect of deriving from the study such instruction as we need seek."[1]

We have been insisting that the materials for the study of logic are to be found mainly in the records which we possess of what thinking has actually accomplished. Our own consciousness, it was said, can supply but a very small quantity of material. To learn what thinking is, one must have as broad a survey as possible of its achievements.

But there is another side to the matter. It must never be forgotten that it is the actual operations of thought with which logic is concerned. The words and propositions which express the results of thinking must never be allowed to take the place of the thoughts themselves. Now, we cannot directly study the thoughts of any other individual. It is only in so far as we interpret, through our own consciousness, the records of what thinking has done, that these records are able to throw any light upon the problem of logic. So in this study, as elsewhere, we must find the key to the material in our own consciousness. If we are to gain any real ideas of the character of the thinking processes by means of which the sciences have been built up, we must reproduce these in our own minds. One's own consciousness must after all furnish the key which makes intelligible the account of the various steps which the thought of mankind has taken in building up science or knowledge.

[1] Whewell, *History of Scientific Ideas*, 3d ed., Vol. I., p. 4.

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The following references may be given in connection with §§ 1 and 2: —

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CHAPTER II

IMPORTANT STAGES IN THE DEVELOPMENT OF LOGIC

§ 5. **The Logic of the Greeks: Aristotle.** — In the fourth and fifth centuries before Christ, a great interest in debate and public controversy sprang up in Athens. There were several reasons for this. In the first place, the Athenians of this period were a very acute and intellectual people; they therefore required some outlet for their mental activities. The various sciences of nature which occupy so much of the thought of the modern world did not exist at that time, nor did the interest exist which was necessary to create them. For although the Greeks of this period had the greatest love and reverence for nature, their interest in natural objects was rather like that of the poet and the artist, than that of the modern man of science; in other words, they were content to enjoy the beauty of natural objects, and to take delight in the harmonies of sound and color which their senses presented to them. They had no desire to pull things to pieces to see how they are made, or to discover the laws according to which they act, and so their mental energy and mental acuteness found its chief outlet in argumentative controversy, and public debating became one of their favourite diversions. The Athenians of those days used to argue, from the pure love of argument, wherever they met, — in the market-

place, in the groves and gardens, and at their meals and banquets.

There was in addition, however, a very practical reason why it was necessary and desirable for one to be able to argue well. A man of property in Athens was constantly exposed to lawsuits, and was obliged to be his own lawyer and defend his cause by pleading before the judges. It was of the utmost practical importance, then, that he should be able to state his cause well, and should be master of all the arts by which the judges would be likely to be influenced. Under these circumstances, it is not difficult to understand why the art of public speaking came to be regarded in Athens as a necessary part of education. And, in response to this demand, there arose a class of teachers called Sophists, who made it their business to instruct young men in all the practical affairs of life, and especially in the art of public speaking, or rhetoric, as it was called. The Sophists do not seem to have made it their object to teach truth to their pupils, or to inculcate in them a love and reverence for truth; they rather sought to make those whom they taught clever men of the world. In teaching the art of argumentation or public speaking they did not seek to point out the methods by which true conclusions could be reached, but rather taught the arts by which the judges could be persuaded, and tricks for the discomfiture of one's adversary. The rhetoric of the Sophists, in other words, was not a science of reasoning, but an art of persuasion and of controversy. It was not necessary to have any real knowledge of the subject under dis-

cussion in order to argue well, but only to be well versed in all the arts of persuasion, and quick to take advantage of the omissions of an opponent.

The theory on which the teaching of the Sophists was based is usually known as scepticism. The Sophists, that is, had come to the conclusion that it is impossible to find any fixed standard of truth. Looking at the diversity of individual opinions and of individual feelings, they declared that knowledge or truth as something objective, or the same for all, is an illusion. Only individual opinions exist; there is no standard by reference to which these opinions may be measured. It is impossible, then, to distinguish false opinions from true. Indeed, the words 'truth' and 'falsehood' can have no real meaning; each individual must be the measure of truth for himself.

Moreover, in the opinion of the Sophists, the same state of things exists with regard to our moral ideas. There is no standard of right and wrong, just as there is no standard of truth and falsehood. Each man has the right to choose what he regards as most advantageous for himself. The traditional rules of morality have no authority over the individual, nor is it possible to discover any rules of morality which are binding on all men. It is the part of wisdom to consult one's own interest in acting, and to seek to secure one's own advantage. Moral distinctions, like logical distinctions, are purely relative and individual.

Socrates was the great opponent of the ethical scepticism of the Sophists. They had concluded, from the diversity of individual opinion on moral questions, that

there is no real or absolute distinction between right and wrong. Socrates, however, was convinced that, if one examined more carefully the nature of the judgments which men pass on matters of right and wrong, one would find common elements or ideas. It is possible, he believed, to find a fixed standard, both in matters of theory and in matters of practice. This common element, however, is not to be discovered in sensation, nor in feelings of pleasure and pain; these are purely individual, and can never serve as a universal standard. But beneath the diversity of sensation and feelings there is the thought, or concept, which is common to all men. When rational beings come to understand each other, they must agree as to the nature of the fundamental virtues, — justice, temperance, courage, etc. It is true that few men have thought about these matters, and are able to express their meaning clearly; but every man, as a rational being, carries these fundamental notions in his mind. Now, in order to refute the moral scepticism of the Sophists (and it was this side of their teaching which Socrates especially opposed), it is necessary that the ethical notions, or concepts, which are implicit in the minds of men shall be drawn out and carefully defined. How is this to be accomplished? Socrates did not undertake to teach men what ideas they should hold regarding the nature of any of the virtues; he rather made them partners in an investigation, and by means of skilful questions tried to assist them in discovering the real nature of goodness for themselves. Another point to be noticed is that the definition of the various virtues was reached

as a result of comparing the views of a number of individuals. In this way, by comparing the opinions of many men, of different professions, and of different grades of society, he was able to separate what was merely individual and relative in these opinions, from what was unchanging and absolute.

Plato, the disciple of Socrates, continued the work of his master. He did not confine his attention wholly to the moral conceptions, but showed that the Socratic method could also be used to refute the intellectual scepticism of the Sophists. In other words, he proved that in the concept, or thought, as opposed to sensation, a standard of truth is to be found, as well as a standard of morality. Knowledge arises from thinking, and it is possible to compare our thoughts, however impossible it may be to find any basis of comparison in our sensations.

Plato's disciple, Aristotle, is of great importance in the history of logic. He undertook a thorough investigation of the process of reasoning, and sought to show what conditions and principles are necessarily involved in reaching certainty. Aristotle was thus the founder of logic, as well as of psychology, zoölogy, and a number of other sciences. His most important logical works are the *Categories*, *De Interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, *and the Sophistical Elenchus*, a treatise on Fallacies. These writings came afterwards to be known as the *Organon* (or scientific instrument) of Aristotle. They contained, in the first place, what we call theory of knowledge (a discussion of the structure of knowledge, and of the scientific principles

upon which it rests), which formed an essential part of Aristotle's philosophical system. But they also furnished the practical application of these principles. In his doctrine of the syllogism, which is found mainly in the *Prior Analytics*, he showed what are the only valid forms of reasoning, and thus furnished the pattern or type to which all proofs must conform. He also classified, in his work on Fallacies, the various species of false reasoning; and showed how false arguments could be refuted and exposed by the principles which he had discovered. The form to which Aristotle maintained that all true reasoning can be reduced was as follows: —

All men are mortal,
Socrates is a man,
Therefore Socrates is mortal.

This is called a **Syllogism**, and it is made up of three propositions. The first two propositions are called **Premises**, and the last the **Conclusion**. Every piece of reasoning, all proof, can be reduced to this form. Of course, the propositions which make up the syllogism do not always stand in this order, and sometimes one of them may be omitted. Thus in the argument: 'he ought to be supported by the state, for he is an old soldier,' the conclusion stands first, and one premise is wanting entirely. It is easy to see, however, that the real argument when properly arranged is equivalent to this: —

All old soldiers ought to be supported by the state,
He is an old soldier,
Therefore he ought to be supported by the state.

Now the part of Aristotle's logic which was best

worked out, was a theory of proof or demonstration by means of the syllogism. Here he showed clearly the various ways in which different kinds of propositions could be combined as premises to yield valid conclusions, and proved that no conclusion could be drawn from other combinations. This part of the Aristotelian logic has come down to us almost unchanged, and is the subject of Part I. of the present volume.

It will be noticed that, in the doctrine of the syllogism, Aristotle was dealing with that kind of reasoning which undertakes to demonstrate the truth of some fact, by showing its relation to a general principle which every one admits. In other words, this part of his work may be called the logic of proof or demonstration. Aristotle was at one time of his life a teacher of rhetoric, and he seemed always to have aimed at putting this art of reasoning on a scientific basis. That is, for the rules of thumb and questionable artifices of the Sophists, he wished to substitute general laws and methods of procedure which were based upon a study of the principles and operations of reason. By complying with the rules which he laid down, an argument will necessarily gain the assent of every rational being.

But we do not employ our reason merely in order to demonstrate to ourselves or to others what we already know. We seek to discover new facts and truths by its aid. In other words, we not only wish to prove what is already known, but also to discover new facts, and we need a logic of Discovery, as well as a logic of Proof. This distinction between proof and discovery corresponds in general to that between Deduction and In-

duction. Deduction is the process of showing how particular facts follow from some general principle which everybody admits, while Induction shows the methods by which general laws are obtained from an observation of particular facts. Now Aristotle, as we have seen, furnished a very complete theory of Deduction, or method of proof. But he did not treat of Induction, or the method of passing from particular facts to general laws, with anything like the same completeness. Moreover, what he did write on this subject received no attention for many centuries. Aristotle was himself a great scientific observer, and may well be regarded as the father of the natural history sciences. But, in his logical writings, his main object seems to have been to present a true theory of argumentation, as opposed to the false theories of the Sophists. Science, too, was only in its beginning when Aristotle wrote, and it was impossible for him to foretell the methods of discovery which it has actually employed.

After Aristotle's death (322 B.C.), and after the loss of Athenian independence, there was a great decline of interest in matters of mere theory which had no direct application to the practical affairs of life. The Stoic school did make some slight additions to logical theory, but like their opponents, the Epicureans, they regarded practice, the art of living well, as the supreme wisdom of life. The Romans, who derived their knowledge of Greek philosophy largely from the Stoics, were also interested in the practical advantages of logic, rather than in its theoretical side. It was the possibility of applying the laws of logic to rhetoric and public speaking

which especially interested Cicero, who was the first to make Latin paraphrases and adaptations of Greek logic in his rhetorical works.

§ 6. **Logic during the Middle Ages.** — For more than seven hundred years, during the Middle Ages, the Greek language and literature was almost unknown in Western Europe. During this time, almost the only sources of information regarding logic were Latin translations of Aristotle's *Categories*, and of an Introduction to the same work by Porphyry, who lived 232–303 A.D. Both of these translations were made by Boethius (470–525), who is best known as the author of *The Consolations of Philosophy*. Even when scholars again became acquainted with the original works of Aristotle, in the latter part of the Middle Ages, they did not really understand their true significance. They took the husk, one may say, and neglected the kernel. They adopted the Aristotelian logic as an external and arbitrary set of rules for the guidance of thinking, and neglected entirely the scientific theory upon which these rules were based. A great deal of ingenuity was also shown in subdividing and analyzing all possible kinds of argument, and giving the particular rule for each case. This process of making distinctions was carried so far that scholastic logic became extremely cumbersome and artificial. Its pretensions, however, rapidly increased; it claimed to furnish a complete instrument of knowledge, and a sure standard for discriminating between truth and falsehood.

It is not very difficult to understand why this set of logical rules

seemed so satisfactory to the age of Scholasticism. The men of this period had no desire to increase their knowledge; they supposed that they were already in possession of everything which was worth knowing. Their only object was to weave this knowledge into a system, to show the connection and interdependence of all its parts, and thus to put it beyond the possibility of attack. And for this purpose, the school logic was admirably adapted; it was always possible to bring every case which could arise under one or other of its rules.

There is no doubt that the Aristotelian logic had a real value of its own, and that it exercised a very important influence upon Western civilization, even in the form in which it was taught by the Schoolmen; but there is, of course, nothing complete or final about it. Its main purpose, as we have already seen, was to furnish a method by means of which the knowledge we already possess may be so arranged as to be absolutely convincing. But the centre of intellectual interest has changed since mediæval times. We are not content merely to exhibit the certainty and demonstrative character of the knowledge which we already have, but we feel that there is a great deal of importance still to be discovered. So that, in modern times, one may say the desire to make discoveries, and so add to the general stock of knowledge, has taken the place of the mediæval ideal of showing that the traditional doctrines taught by the church are absolutely certain and convincing. And when men became conscious of the importance of gaining new knowledge, and especially knowledge about nature, they at once saw the necessity for a new logic, or doctrine of method, to aid them in the undertaking.

§ 7. **The Logic of Bacon.** — All the great thinkers of the sixteenth and seventeenth centuries saw clearly that the school logic is simply a method of showing the certainty of the knowledge we already possess, and does not aid us at all in making new discoveries. A new method, they all declared, was an absolute necessity. The new point of view was put most clearly and eloquently by the famous Francis Bacon (1561–1626), at one time Lord Chancellor of England. Bacon called his work on logic the *Novum Organum*, thus contrasting it with the *Organon*, or logical treatises of Aristotle. An alternative title of the work is, *True Suggestions for the Interpretation of Nature.* Bacon begins this work by showing the advantages to be gained from a knowledge of nature. It is man's true business, he tells us, to be the minister and interpreter of nature, for it is only by becoming acquainted with the laws of nature that we are ever able to take advantage of them for our own ends. "Knowledge and human power are synonymous, since ignorance of the cause prevents us from taking advantage of the effect." The discovery of the laws of nature, which is therefore of so great practical importance, cannot be left to chance, but must be guided by a scientific method. And it is such a method which Bacon endeavours to supply in the *Novum Organum.*

The method which Bacon proposed seems to us very simple. If we would gain new knowledge regarding nature, he says, and regarding natural laws, we must go to nature herself and observe her ways of acting. Facts about nature cannot be discovered from logical propositions, or from syllogisms; if we would know the

law of any class of phenomena, we must observe the particular facts carefully and systematically. It will often be necessary, also, to put pointed questions to nature by such experiments as will force her to give us the information we want. Knowledge, then, must begin with observation of particular facts; and only after we have made a great number of particular observations, and have carefully classified and arranged them, taking account of all the negative cases, are we able to discover in them the general law. No hypotheses or guesses are to be made; but we must wait until the tabulations of the particular phenomena reveal the general 'form' or principle which belong to them all.

It will be frequently necessary to refer to Bacon's work in what follows. At present, it is sufficient to note that Bacon showed that a knowledge of nature cannot be attained through general propositions and logical arguments, but that it is necessary to begin with the observation of particular facts. He emphasized, also, the importance of systematic observation and carefully planned experiments, and showed that knowledge must begin with facts of perception. This is the method of induction, and Bacon is usually said to have been the founder of the inductive sciences of nature.

§ 8. **Logic since the Time of Bacon.** — Another and quite different method of extending knowledge was proposed by the great Frenchman, Descartes (1596–1650), who took mathematics as the type to which all knowledge should conform. That is, he supposed that the

true method of extending knowledge was to begin with general principles, whose truth could not be doubted, and to reason from them to the necessary character of particular facts. Descartes and his followers thought that it was possible to discover certain axiomatic propositions from which all truth could be derived through reason. They thus emphasized Deduction rather than Induction, and reasoning rather than observation and experiment. The spirit of Bacon's teaching was, however, continued in England by John Locke, in the *Essay Concerning Human Understanding* (1690). During the next centuries, philosophical thinkers were divided into two great schools, — Rationalists, or those who agreed in the main with Descartes, and Empiricists, or Sensationalists, who followed the teachings of Bacon and Locke.

Although the natural sciences made great advances during the seventeenth and eighteenth centuries, there seems to have been no effort made to analyze and describe the methods which were actually being employed. In England, at least, it seems to have been assumed that all discoveries were made by the use of the rules and methods of Bacon. One of the first writers to attempt to explain the method used by the natural sciences was Sir John Herschel (1792–1871). His work, *Discourse on the Study of Natural Philosophy*, was published in 1832. A little later, and with the same object in view, William Whewell (1794–1866), afterwards Master of Trinity College, Cambridge, undertook his *History of the Inductive Sciences*, which was followed some time after by the *Philosophy of the*

Inductive Sciences. The man, however, who did most towards putting the study of logic on a new basis was John Stuart Mill (1806–1873), the first edition of whose *Logic* appeared in 1843. We shall have frequent occasion to refer to this work in future discussions. It is sufficient to say here that Mill continues the empirical tradition of the earlier English writers in his general philosophical position. Mill's book gave a great impulse to the study of logic. Before it was published, writers on the subject had confined their attention almost exclusively to the syllogistic or deductive reasoning. Mill, however, emphasized strongly the importance of induction; indeed, he regarded induction as the only means of arriving at new truth, deduction being merely a means of systematizing and arranging what we already know. Though few logicians of the present day adopt this extreme view, the importance of inductive methods of reasoning, and the necessity of studying them, have now become generally recognized. Most modern writers on logic devote a considerable amount of attention to induction. The reader will find that Part II. of the present volume deals with this subject.

There is still another side of logic which has been developed in the English-speaking world since the time of Mill, though it is a direct continuation of the movement started in Germany by Kant more than a hundred years ago. The so-called 'modern' logic has laid aside the formalism and paradoxical mode of expression adopted by Hegel, but the fundamental conceptions with which it works are essentially the same as those

employed by the latter in his *Wissenschaft der Logik* (1816–1818). It has been within the last twenty years that the results of German idealism — the doctrines of Kant, Fichte, Schelling, and Hegel — have become naturalized in England and America. And largely as a consequence of these teachings, a new conception of the nature of thought has grown up, and given rise to investigations which may be said to have created a 'modern' logic that is fairly entitled to rank beside its sister science, the 'new' psychology.

The Aristotelian doctrine of the syllogism is a purely formal science. In the form in which it is represented in ordinary text-books, it might perhaps be more properly described as the art of arranging our knowledge in such a way as to compel assent. The 'matter' with which thought is supposed to work is supplied to it in form of concepts and judgments. The problem which formal logic has to solve is to define and classify the various kinds of concepts with which thought operates, and to determine the various relations in which these stand when combined into judgments. Similarly, it has to show what combinations of judgments can be employed as premises leading to valid conclusions in the syllogism. The criterion of truth employed in these investigations is the principle of non-contradiction or consistency. Inconsistent combinations of concepts, that is, are ruled out; but so far as the doctrine of the syllogism goes, anything is true which is not self-contradictory.

Now, without questioning the practical value of its canons, it is obvious that formal or syllogistic logic does

not take any account of many of the processes of everyday thought, and that its rules go but a little way in helping us to distinguish the true from the false. For, in the first place, to think is not merely to combine and arrange ideas already in our possession. This might enable us to render clearer and more definite what we already know, but would never enable us to gain new knowledge. The real movement of thought — as opposed to its merely formal procedure — consists in the formation of new ideas and new knowledge through actual contact with the world of experience. A complete account of the intellectual process, then, must deal with the relation of the mind to objects; it must investigate the various activities by means of which thought interprets the world and builds up the various sciences of nature and of man.

The recognition of the importance of induction, and of the necessity of studying the methods of the inductive sciences which was brought about by Whewell, Mill, and others, was a step in the right direction, for it called attention to a kind of thinking which occupies a large place in our intellectual life, and also gave rise to a truer conception of the nature of thought itself. But even Mill did not reach the idea which guides modern logicians, that thought or intelligence is one from beginning to end, and that the various logical processes are all parts of one whole, or rather ways in which intelligence operates in different circumstances, or at different stages of its development. He still treats of logical processes, like conception, judgment, and reasoning, as if they were quite separate from

each other; and, as has already been noticed, in his zeal for induction, he fails completely to do justice to syllogistic reasoning.

As opposed to the division of mind into separate faculties, the thought by which modern logic is dominated is that of the unity and continuity of all intellectual life. Thought is regarded as an organic, living function or activity, which remains identical with itself throughout all its developing forms and phases. The problem, accordingly, which logic must set before itself is to show the unity and interrelation of all of the intellectual processes. No one of the steps or stages in this process can be completely understood when viewed by itself: each is what it is only in and through its connection with the whole of which it forms a part. No hard and fast boundary lines are to be drawn between the different stages of the reasoning process, but it must be shown that the whole nature of intelligence is involved more or less explicitly at each step. So far only the broad outlines of this theory have been filled in; but the conception of an organism whose parts are developing in mutual relation and interdependence, promises to be as fruitful when applied to logic as it has already shown itself to be in the other sciences.

Besides the ordinary histories of philosophy the reader may consult for the history of logic: Prantl, *Geschichte der Logik im Abendlande*, 4 vols., Leipsic, 1855–1870; which extends, however, only to the close of the mediæval period. Harms, *Geschichte der Logik*, Berlin, 1881. Ueberweg, *System der Logik*, 4th ed., 1874; Eng. trans. of 3d ed., London, 1874. Adamson, article 'Logic,' in the

Encyl. Brit., 9th ed. Sir William Hamilton's *Lectures on Logic*, also contain much historical information.

Among modern works on logic, the following may be mentioned: J. S. Mill, *A System of Logic*, London, 1st ed., 1843; 9th ed., 1875. W. S. Jevons, *The Principles of Science*, London, 1874; 2d ed., 1877. Also, by the same author, *Studies in Deductive Logic*, 1880; and *Pure Logic*, 1890. H. Lotze, *Logik*, 1874; Eng. trans., London, 1881 and 1888. W. Wundt, *Logik*, 2d ed., 1896. C. Sigwart, *Logik*, 2d ed., 1889-1893; Eng. trans., London and New York, 1895.

The newer development of logic is well represented by F. H. Bradley, *The Principles of Logic*, London, 1886. B. Bosanquet, *Logic, or the Morphology of Knowledge*, London, 1888; and *The Essentials of Logic*, London and New York, 1895. L. T. Hobhouse, *The Theory of Knowledge*, London, 1896, may also be mentioned in the same group of writers, although he has been, perhaps, more influenced by Mill than by any other writer.

The following works, among others, have proved useful as text-books: W. S. Jevons, *Elementary Lessons in Logic*, London and New York, 1870. A. Bain, *Logic, Deductive and Inductive*, New York, 1883. J. H. Hyslop, *The Elements of Logic*, New York, 1892. W. Minto, *Logic Inductive and Deductive*, New York, 1894. J. G. Hibben, *Inductive Logic*, New York, 1896.

PART I.—THE SYLLOGISM

CHAPTER III

THE SYLLOGISM AND ITS PARTS

§ 9. **The Nature of the Syllogism.**—The theory of the syllogism, as has been already stated (§ 5), was first worked out by Aristotle. And it stands to-day in almost the same form in which he left it. A few additions have been made at different points, but these do not affect materially the main doctrine. In dealing with the nature of the syllogism, we shall first try to understand its general aim and purpose, or the results which it seeks to bring about. We shall then have to analyze it into the parts of which it is composed, and to examine and classify the nature of these elements. Finally, it will be necessary to discover what rules must be observed in order to obtain valid conclusions, and to point out the conditions which most commonly give rise to error or fallacy.

In the first place, it is to be noticed that syllogistic logic deals with the results of thinking, rather than with the nature of the thought-process. Its object is not to give an account of the way in which thinking goes on, but to show how the ideas and thoughts which we already possess may be combined so as to compel

assent. The ideas which it uses as material are fixed by having been expressed in language. Indeed, it is largely with words, as the expression of thoughts, that syllogistic logic deals. Many of the discussions with which it is occupied have reference to the meanings of words and propositions; and the rules which it furnishes may be taken as directions for putting together propositions in such a way as to lead to a valid conclusion. Nevertheless, it is important to remember that these rules are not arbitrary and external, but find their justification in the nature of thought. Indeed, the theory of the syllogism, when rightly understood, may be said to reveal the fundamental characteristics of the process of intelligence. For it brings together facts in such a way as to make evident their relation and dependence. It connects a judgment with the grounds or reasons which support it, and is thus a process of systematization. In order to understand the significance of the rules of syllogistic logic, then, it will frequently be necessary to look beyond words and propositions to the act of thought whose result they express.

A great deal has been written regarding the principles, or laws of thought, which are employed in syllogistic reasoning. It seems better, however, to postpone the definite consideration of this subject until the student has learned more about the various kinds of syllogisms, and has had some practice in working examples. In dealing with the nature and principles of thought in the third part of this book, it will be necessary to discuss this question at length. Even at the present stage of

our inquiry, however, it is important to notice that syllogistic reasoning presupposes certain simple and fundamental principles of thought whose nature we shall have to examine hereafter. In particular, the regular syllogism is founded on a principle which we may call the law of Identity, or the law of Contradiction, according as it is stated affirmatively or negatively. Stated affirmatively, this so-called 'law' simply expresses the fact that every term and idea which we use in our reasonings must remain what it is. A is A, or has the same value and meaning wherever employed. The law of Contradiction expresses the same thing in negative language. A cannot be both B and not B. If any term is taken to be the same as another in one connection, it must always be taken to be so; if it is different, this relation must everywhere be maintained. The data or materials which are employed in the syllogism are ideas whose meaning is supposed to be permanently fixed, and expressed in words which have been carefully defined. It would be impossible to reason, or to determine the relation of our ideas, if their meaning were to change without notice, or if the words by means of which they are expressed were used now in one sense, and now in another. It is of course true that our ideas regarding the nature of things change from time to time. And, as is evident from one's own experience, as well as from the history of language, a corresponding change takes place in the meaning of words. But the assumption upon which syllogistic reasoning proceeds, is that the ideas which are to be compared are fixed for the mean time, and that the

words by which they are expressed are used in the same sense throughout the course of the argument. In this kind of reasoning, then, just as in geometry, it is essential that the terms which enter into the argument be clearly and precisely defined, and that when thus determined they shall be taken as fixed and unchangeable until further notice is given.

It is quite possible that all the requirements of the syllogism may be met without its conclusions being true of reality. In other words, an argument may be *formally* true, but really false. It is not difficult to understand why this may happen. The syllogism accepts the ideas and judgments which it compares without criticism. These data are of course the product of previous acts of thinking. But in proceeding to arrange them in syllogistic form, we do not inquire whether or not they are true; *i.e.* adequate to express the nature of the things for which they stand. For the purposes of the syllogism it is only essential that their meanings be clearly understood, and that these meanings be regarded as fixed and permanent.

§ 10. **The Parts of a Syllogism.** — The syllogism may be said to express a single comprehensive act of thought. We may define inference as a judgment which has been expanded so as to exhibit the reasons by which it is supported. In the syllogism

> The geranium has five pointed sepals,
> This plant has not five sepals,
> Therefore it is not a geranium.

we may say that we have the judgment, 'this plant is

not a geranium,' supported by the propositions which precede it, and that the whole syllogism taken together expresses a single thought, which is complete and self-sufficient. It is possible, however, even when one is dealing directly with the process of thinking, to distinguish in it different subordinate steps, various stages which serve as resting places, in the course of its passage to the complete and comprehensive form represented by the syllogism. But it is usual, in dealing with the syllogism, to take a more external view of its nature, and to regard it primarily as made up of words and propositions.

In this sense, a syllogism can, of course, be divided into parts. In the first place, it is composed of three propositions. In the example given above the two propositions which stand first are called the **premises**, since they furnish the grounds or reasons for the proposition which stands last, and which is known as the **conclusion**. However, it is not true that we always find the two premises and the conclusion arranged in this regular order in syllogistic arguments. Oftentimes the conclusion is given first. Frequently, too, one of the premises is not expressed, and has to be supplied in order to complete the argument. Thus the statement, 'he must be more than sixteen years of age, for he attends the university,' is an incomplete syllogism. The conclusion, as will be readily seen, stands first. There is also only one premise expressed. To put this statement in the regular syllogistic form we have to supply the missing premise and arrange it as follows: —

All students of the university are more than sixteen years of age,
He is a student of the university,
Therefore he is more than sixteen years of age.

When one premise of an argument is lacking, the name of *enthymeme* ~~is applied to it.~~ When an argument is defective in this way, it must be remembered that the missing proposition is to be regarded as in consciousness, though not expressed. It is of great importance to form the habit of making clear to oneself the premises by which any conclusion claims to be supported. In this way groundless assumptions are often brought to light, and the weakness of an argument exposed. Whenever words like 'therefore,' 'for,' 'because,' 'it follows,' etc., are used in their proper signification, it is possible to find an argument composed of two premises and a conclusion. But one must not allow oneself to be imposed upon by the mere words, but must insist on understanding exactly what are the premises in the case, and how the conclusion follows from them.

It is possible to carry the division of a syllogism still further. Every logical proposition may be divided into two **terms**, and a **copula** or connecting link. The terms, which are the extremes of the proposition, are named the subject and the predicate. Thus in the proposition, 'the fields are covered with snow,' 'the fields' is the subject, 'are,' the copula, and, 'covered with snow,' the predicate. To reduce a proposition to the logical form in which it is most conveniently treated, it is necessary to express it in such a way that the two terms are united by some part of the verb 'to be,' preferably 'is' or 'are.' Thus the sentence, 'No plant can grow with-

out light and heat,' would be expressed as a logical proposition in the following, or some similar, form: 'No plant is an organism which can grow without light and heat.' 'Men have strong passions,' may be written, 'Men are beings having strong passions.' It is always well to reduce a sentence to some such form, by substituting for the verb of predication some part of the verb 'to be.'

The analysis of the syllogism gives us the divisions under which it is convenient to treat this part of logic. We shall accordingly deal (1) with Terms, (2) with Propositions, and (3) with the Syllogism as a whole.

These divisions, however, are only made for the sake of convenience in treatment. It must not be forgotten that a term is a part of a proposition. To understand the nature of a term, it is necessary to consider the part which it plays in the judgment which the proposition expresses. In other words, the *function* of the term, rather than the form of the word or words employed, must be considered. It is, of course, true that we naturally and commonly use certain word forms to express certain kinds of ideas, just as in the grammatical sentence the different 'parts of speech'—nouns, verbs, etc.,—have each a definite and comparatively permanent function. But even in the sentence, it is the part which the word in its grammatical function plays, rather than its form, which determines whether it is to be classified as a noun or an adjective, a preposition or a conjunction. In dealing separately with terms, as we propose to do in the next chapter, we shall be occupied to a large extent with the *form of words* in which cer-

tain kinds of ideas are usually expressed. But, as the same word or group of words may be used for different purposes, it will be necessary, in order to understand the meaning of terms, to refer frequently to the various ways in which they are used in a proposition.

The same difficulty exists when propositions are considered by themselves, the relation to the complete argument of which they form a part being thus ignored. In this case, however, the results of the isolation are not so apparent, for a proposition forms, in a certain sense, a whole by itself. It is the expression of a judgment which, as we shall see later, is the unitary process of thought. It has thus a significance of its own, as expressing a more or less complete and independent act of thought. Nevertheless, it must not be forgotten that its independence and completeness are only partial and relative. A single proposition cannot stand alone. Taken strictly by itself, a proposition is only a fragment. In order to make it intelligible, it must be brought into relation with the other propositions which state the grounds or reasons upon which it rests, or the conclusion which it helps to support. The logical nature of a proposition will, therefore, depend upon its function in an argument, and in treating of propositions this fact must not be forgotten.

§ 11. **The Proposed Division of Mental Operations.**— It is frequently stated in text-books on logic that corresponding to the division into Terms, Propositions, and Syllogisms, there must be a division of the different kinds of thought, or of operations of the mind. These differ-

ent operations are usually called Simple Apprehension, Judgment, and Reasoning. "The first of these, Simple Apprehension, is the act of mind by which we merely become aware of something, or have a notion, idea, or impression of it brought into the mind. The adjective *simple* means apart from other things, and *apprehension*, the taking hold by the mind. Thus the name or term 'iron' instantaneously makes the mind think of a very strong and very useful metal, but does not tell us anything about it, or compare it with anything else."[1] Judgment, the account continues, is an entirely different action of mind, and comes later than Simple Apprehension. It consists in comparing two notions or ideas derived from simple apprehension in order to ascertain whether they agree or differ. In order to judge, we must have two notions or ideas ready in the mind. The judgment results from comparing these, and affirming that they agree or do not agree. In the same way, having already made judgments, we can combine them into arguments or processes of reasoning by a new and still different activity of mind. Apprehension, judgment, and reasoning are thus supposed to be separate and distinct mental operations. It is true that the later forms employ as their material the finished products of the earlier. But from this point of view, apprehension, judgment, and reasoning simply succeed one another. The real unity which belongs to these operations as forms of intelligence is not set forth.

[1] Jevons, *Lessons on Logic*, pp. 11, 12.

The whole of Part III. of the present book may be regarded as an argument against this point of view. We shall there endeavour to show that thinking is not a process of externally joining on part to part, but consists in a development or expansion of knowledge from within. And, in particular, we shall try to exhibit the essential unity of intellectual processes by whatever name they may be called, and at whatever stage of development they may be found. Without anticipating too far our future discussions, we may point out that the primary process of thought is not 'Simple Apprehension,' but Judgment. In other words, it is impossible to apprehend or passively receive ideas. 'To get an idea,' or to understand the meaning of a term, is only possible when the mind judges or interprets things for itself. To have an idea or concept of anything, then, is to be able to judge more or less clearly and confidently regarding it. I have an idea of 'iron' when I judge that it is 'black' and 'heavy' and 'malleable.' And the more complete and exact we can make our judgments, the better is the idea or apprehension which we obtain of the thing in question. Intelligence or thought must not be regarded as at first merely receptive. It does not begin by laying hold of separate ideas or terms, and afterwards call in judgment as a new kind of process to bring the former into relation. But it is from the first a systematizing and relating activity which proceeds from the less perfect to the more perfect form of judgment (*cf.* §§ 73, 74).

CHAPTER IV

THE VARIOUS KINDS OF TERMS

§ 12. **Singular, General, and Collective Terms.** — A logical term, as we have already seen, is an element of a proposition. In dealing with terms apart from propositions, we shall be concerned mainly with different classes of words and the meanings which they usually express. It will be impossible, however, to fix the meanings of terms absolutely without reference to the way in which they are used in propositions. The first division which we have to notice is that into Singular or Individual, General, and Collective terms.

(1) A **Singular** or **Individual** term is one which can be applied in the same sense to but a single thing. The main purpose of Singular terms is to refer to, or identify, some individual object. Proper names are all singular. It is true that proper names are sometimes used to denote a class of objects, as, *e.g.*, 'a Daniel,' 'a Mephistopheles.' But when thus employed they lose their real character as proper names. That is, their function is no longer merely to identify certain individuals by *naming* them, but to *describe* them by mentioning certain qualities or characteristics which they are supposed to possess. But the ordinary purpose in using a proper name is to indicate some individual to whom the name belongs. In this sense, then, proper names are Singular.

In addition, any word or group of words which is applied to a single thing may be regarded as singular. And by 'single thing,' we mean anything which is thought of as one, as well as objects which are perceived through the senses. Thus, 'the waterfall just below the bridge,' 'the centre of the earth,' are singular terms, and so also are words like 'justice,' 'goodness,' 'the chief end of man.' It is perhaps more doubtful whether we should call terms such as 'whiteness,' 'sweetness,' singular, since we speak of different degrees and kinds of whiteness and sweetness. The question would have to be decided in every case by reference to the way in which the terms are employed in propositions.

(2) A **General** term is a name which applies to a whole group of objects. It is not limited, like the singular name, to a single thing, but applies to a number of different things. All class names like 'metal,' 'man,' 'works on logic,' are of this character. The general name belongs to each and every individual of a whole class. Thus iron, gold, silver, etc., are 'metals'; and A, B, and C, 'men.'

(3) A **Collective** term, on the other hand, is a name applied to a number of individuals when taken together and treated as a whole, as 'an army,' 'an audience.' It is important to distinguish carefully between general and collective terms. A general term is a name which applies equally to each individual of the group; or, in other words, it is used of the individuals *distributively*. A collective name belongs to the whole, but not to the separate parts of the whole. Thus we say that 'sol-

dier' is a general name, and is used distributively of each man in a regiment. 'Regiment,' however, is a collective name, for it applies only to the whole group, and not to the individual soldiers.

Ambiguity sometimes arises from the fact that the English word 'all' is used in both of these senses. That is, it may mean 'all taken together,' or 'each and every.' Thus we can say: 'All the angles of a triangle are less than two right angles'; and 'all the angles of a triangle are equal to two right angles.' In the former sentence, the word 'all' is used distributively; in the latter, collectively. In Latin two different words are used: *cuncti* expresses the collective sense of 'all,' and *omnes* its distributive signification.

It is worth noticing in this connection that it is the *use* which is made of terms, rather than the *form* of the words composing them, which determines their logical character. Thus terms which are collective in one connection may be general in another. 'Regiment,' for example, is a collective term with reference to the soldiers which compose it, but general when used as a common term for a number of similar divisions of an army. The same is also true of terms like 'grove,' 'mob,' 'class,' etc. Again, collective terms may be very properly regarded as singular when the proposition in which they are used emphasizes the unity and solidarity of the group. A proper name is sometimes applied to a collection of individuals that are permanently united or that have acted together on some historic occasion, as, for example, 'The Fifth Cavalry regiment,' 'The Charge of the Six Hundred.'

§ 13. **Abstract and Concrete Terms.** — Terms are further divided into *abstract* and *concrete* terms. The word 'abstract' is often used popularly to describe anything which is difficult to understand. Etymologi-

cally, it signifies drawn off, separated (*abstraho*, to draw off, take away). We may distinguish two senses in which the word is used, both, however, being derived from its etymological signification.

(1) A term is called *abstract* when it refers to some object which cannot be directly perceived through the senses, and *concrete* when such perception is possible. Thus 'a beech tree,' 'a tall man,' 'a sweet taste,' being names of things which can be perceived, are concrete. Words like 'sweetness,' 'hardness,' etc., have no objects of sense directly corresponding to them, and are for this reason called abstract. The same is true of terms like 'individuality,' 'equality,' 'justice,' etc. These words represent objects of thought, rather than objects of sense. There may be cases or instances of 'equality,' 'justice,' etc., which fall under our perception, but the real object to which these words correspond is not a thing which can be perceived through the senses at all. Their reality is conceptual, or for thought, not something directly revealed through the senses.

It is important to notice that there are degrees of abstractness in terms, according as the objects for which they stand are nearer to, or further removed from ordinary sense-perception. All general or class names are abstract. One cannot point to a single object, to which the term 'metal,' for example, or the term 'man' corresponds. But although such terms have no direct sensuous object, yet we feel that they stand nearer to sense-perception, and are therefore less abstract, than words like 'animal,' 'inorganic substance.' These terms, again, are perhaps less abstract than 'energy,' or 'spirit,' or even than singular terms like 'justice,' 'the ground of the universe,' etc.

(2) Again, the word 'abstract' is applied to any object which is treated apart from the whole to which it belongs. Thus it would be an abstraction to attempt to represent the nature of a leaf in complete isolation from the plant to which it belongs, or to consider the nature of a man without regard to the social institutions — family, church, state, etc. — of which he is a member. Of course, it is essential when dealing with a complex whole to analyze it into its parts, and to understand just what is the nature of each part when taken by itself. But in order to comprehend fully the nature of the parts, it is necessary to restore them to their proper setting, and to see their relation to the concrete whole. In this sense of the word, then, 'abstract' applies to what is taken out of its proper setting, broken off, and considered apart from the things to which it is organically related. Concrete, on the other hand, means what is whole and complete, a system of things which mutually support and explain one another.

Since science has to analyze things into their elements, and to investigate and describe these elements in detail, it is impossible entirely to avoid abstraction. But it is necessary, in order to completely understand the nature of a complex object, that the abstractions of analysis shall be corrected. In other words, the concrete relations in which things stand must not be ignored in investigating them. The conception of evolution in recent times has done much to render the biological sciences more concrete in the sense in which we are now using the term. For it has substituted, for the old method of treating each species of plant and animal as

distinct and separate, 'cut off from each other as if by a hatchet,' the view that all organic beings are members of one family, and can be properly understood only in their relations to one another.

It is interesting to notice that, from this point of view, sense-perception is more abstract than thought. For the senses represent things in isolation from each other. Each thing is known in sense-perception as a separate individual, occupying its own space and time, and in this way, cut off from its fellows. It is the business of thought, on the other hand, to discover the relations between things, and the principles according to which they are united. Thinking thus overcomes the abstract point of view of sense-perception by showing that what appear to the latter as separate objects are really closely and necessarily connected as members of a common unity or system. Each science takes as its province certain facts which resemble one another, but which nevertheless appear to sense-perception to be quite independent. It attempts by thinking to bring these facts into relation, to show that they are all cases of some law, that there is a common principle which unites them as parts of a whole or system. The law of gravitation, for example, expresses the unity which thought has discovered in things which appear to sense-perception as different as the falling of an apple, the movements of the heavenly bodies, and the ebb and flow of the tides. Scientific knowledge, then, is more concrete than the facts which we learn from ordinary sense-perception, because it brings to light real unity and connection in facts which appear to be entirely isolated and independent from the latter point of view.

In employing the terms 'Abstract' and 'Concrete' it is of the utmost importance to distinguish the two significations of the words. From one point of view, as we have seen, all thought terms are abstract, as opposed to words which refer directly to objects of sense-perception.

In another sense, 'abstract' denotes what is partial and incomplete, what is taken by itself and out of relation to the system of things to which it belongs. And since the real connection and relations of things are not given by perception, but have to be discovered by thought, the knowledge which the latter yields is more concrete, in this latter sense of the term, than that afforded by the former.

§ 14. **Positive and Negative Terms.** — The distinction between Positive and Negative terms is very obvious. **Positive** terms express the existence of some quality, or group of qualities, in the objects which they denote; as, *e.g.*, 'happy,' 'good,' 'equality,' 'organism,' etc. A **Negative** term, on the other hand, indicates the absence of qualities or properties in some object; 'bad,' 'unhappy,' 'inorganic,' 'injustice,' for example, are negative terms. Negative terms are often formed from positive by means of the affix, *less*, as in 'hopeless,' or by means of certain prefixes, of which the more common are *un*, *in*, *dis*, *a*, *anti*. Words which are positive in form are, however, often negative in meaning, and are used as the contradictories of other terms. Thus 'ignorant' is generally regarded as the negative of 'learned,' 'darkness' is the negative of 'light,' etc. It is not always possible, however, to find a separate word to express the exact opposite of every positive term. Words are used primarily to express the presence of qualities, and the negative idea may not be referred to so frequently as to require a separate word to express it. Thus there is no independent term to express the opposite of 'trans-

ferable,' but by employing 'not' as a negative prefix we obtain 'not-transferable.'

It is always advisable when we wish to limit a term strictly to its negative application to employ *not* or *non* as a prefix. Words which are negative in form frequently have a more or less definite positive signification. Jevons points out that words like 'unloosed' and 'invaluable,' though negative in form, have a positive meaning. But, in addition, terms like 'unhappy,' 'immoral,' do not merely indicate the absence of positive qualities, but also express some positive properties of the objects to which they are applied. We speak of a person 'being positively unhappy'; and we employ 'non-moral' to express the simple negative relation rather than 'immoral.'

On the other hand, there are certain terms which are positive in form that express the absence of qualities or attributes. Words like 'blind,' 'dumb,' 'maimed,' 'orphaned,' may be given as examples. These are often called Privative terms, rather than Negative, the distinction being that they refer to qualities or attributes which the objects to which they are applied naturally and usually have, but of which they have been deprived, or which they have never possessed. Thus 'blind,' as applied to a man, implies that he has lost or is destitute of the ability to see which naturally belongs to a human being.

Again, other terms seem to be positive and negative solely in relation to each other. 'Element' and 'compound' are related as negatives or contradictories. It is difficult, however, to say which term is in itself negative or positive.

It is important to notice the distinction between the relation in which positive and negative terms stand to each other, and that expressed by words which have to do with opposite extremes of something which possesses quality or degree. Positive and negative terms are mutually **contradictory**. An element is what is *not* a compound, 'dishonest' is the contradictory of 'honest,'

and as contradictories there is no middle ground between them. What is not an element, is a non-element or a compound. **Opposite or contrary** terms, on the other hand, express a great difference of degree in the objects to which they refer. Thus 'foolish' is the opposite of 'wise,' 'cold' the opposite of 'hot,' and 'bitter' of 'sweet.' But there is always the possibility of a middle ground between opposites. We cannot say that a man must be either wise or foolish, a taste either sweet or bitter. The logical contradictory of 'wise' is 'not-wise,' of 'bitter,' is 'not-bitter,' etc. Opposite or contrary terms, then, must be carefully distinguished from contradictories.

§ 15. **Absolute and Relative Terms.** — Another classification of terms, which is usually given by logicians, is that into absolute and relative terms. An **absolute term** is one which refers to an object which exists by itself, and has an intelligible meaning when taken alone. Thus, 'tree,' 'house,' 'the State of New York,' are examples of absolute terms. A **relative term**, on the contrary, is a name which only derives a meaning from its relation to something else. The term 'parent,' for example, cannot be thought of except in relation to 'child.' Similarly, 'teacher' is relative to 'pupil,' and 'cause' to 'effect.' Relative terms usually go in pairs and are known as **Correlatives.** Adjectives, as well as nouns, may be related in this way. The presence of one quality or characteristic in a thing frequently implies the presence of others. Thus, ignorance and superstition, sympathy and tolerance, are necessary correla-

tives, because the one involves the other, or is invariably connected with it.

It is of course true that no finite thing is completely absolute or independent of other things. The nature of everything is largely determined by the nature of the other things with which it stands in relation. A tree, for example, is relative to the seed from which it sprang, the soil in which it grew, the sunshine, rain, etc., which accompanied its growth. All finite things have a beginning and an end, and are also influenced throughout the whole period of their lives by the action of other things. They are therefore not completely absolute or independent. It is, however, possible to make a distinction between words which are the names of things that are comparatively independent, and may for ordinary purposes be considered by themselves, and those which have only a meaning when regarded as correlatives.

§ 16. **Extension and Intension of Terms.** — In the foregoing sections of this chapter we have explained the nature of the various kinds of terms with which logic deals. It is now necessary to notice two different purposes for which terms are employed. In the first place, terms are used to refer to things, to name and identify them. Thus 'man' refers to the different individual men, John Smith, Thomas Brown, etc., as well as to the various classes of men, Caucasians, Indians, Mongolians, etc. As denoting or naming objects, whether these be individual things or classes of things, terms are said to be employed in Extension. But words are also used to describe as well as to name. That is, they represent the qualities or attributes belonging to things for which they stand. They are not bare names without signification, but as the expression

of ideas they stand for certain qualities or characteristics which things are judged to possess. 'Man,' for example, is not merely a name which may be applied to individual human beings or races of men, but it implies that the objects so named have certain qualities, such as animal life, reason, and the power of communicating with their fellows. When words are used in this way to define or describe things, rather than merely to name them, they are said to be employed in Intension.

The terms 'Denotation' and 'Connotation' were used by Mill instead of Extension and Intension, respectively, and have been adopted pretty generally since his time. To 'denote,' is to point out or specify the objects for which a term stands; and to 'connote' is to take account of the attributes or qualities which a name implies. The words 'breadth,' and 'comprehension,' are also sometimes used as synonymous with Extension, and 'depth,' or 'content,' instead of Intension. The terms to be remembered, however, are Extension or Denotation, and Intension or Connotation.

It is useful to accustom ourselves to distinguish these two functions or uses of a term, — to notice, that is, the things or classes of things to which the name applies, — and also to reflect upon the signification, or ways of judging about these things, for which the name stands. The Extension of a term, as has been said, indicates the objects to which a name applies, and the Intension the qualities or attributes which it signifies. From the point of view of extension, therefore, 'planet' may be defined by mentioning the names of the various planets, Mercury, Venus, the Earth, Mars, etc. Similarly, a term like 'carnivora' might be given in extension by nam-

ing seals, bears, weasels, dogs, wolves, cats, lions, etc. Usually, however, we define from the point of view of intension, that is, by stating the qualities or characteristics for which the term stands. Thus we give the intensive meaning of 'planet,' as a heavenly body which revolves in an elliptical orbit round the sun. 'Carnivora,' defined from the same point of view, are mammalian vertebrates which feed upon flesh. It is not unusual, however, to supplement an intensive definition by turning to extension and enumerating examples. Thus we might add to the definition of 'carnivora' just given, the words, 'as lions, tigers, dogs, etc.'

It is sometimes said that the intension and extension of terms vary inversely. This is simply an attempt to give a mathematical form of statement to the fact that the more a term is defined, or limited, by the addition of attributes, the fewer are the objects to which it applies. 'As the intension of a term is increased its extension is diminished, and *vice versa*,' is the form in which the relation is often stated. For example, let us begin with some class-name like 'animal,' which has a great extension, and add a new attribute, 'rational.' We get 'rational animal' = man. This term now applies to a much smaller number of individuals than 'animal.' The extension of the former term has been diminished, that is, by increasing the intension. If we add to 'man' still another attribute like 'white,' we again lessen the number of individuals to which the term applies. In general, then, it can be seen that the extension of a term is lessened as it is made more definite by the addition of new attributes. And, conversely, by stripping off

attributes, by 'decreasing the intension,' the number of individuals to which a term applies is increased. There is, however, no exact ratio between the increase or decrease of intension and the corresponding change in extension. Indeed, the extension of a class may increase greatly without any loss of intension on the part of the term by which the idea is expressed. Thus the meaning or intension of the term 'man' has not lost, but rather gained, during the last hundred years by the increase of population throughout the world.

Extension and intension, according to the view just given, represent two different uses or functions of terms. Every term denotes some object or group of objects more or less directly, and at the same time connotes or signifies certain qualities or attributes. Sometimes the one purpose, sometimes the other, is the predominant one. Proper names, for example, are used primarily to denote or mark out things, and do not directly qualify or describe them. In the proposition, 'these animals are all vertebrates,' the predicate term 'vertebrates' is employed less as a name of a number of animals, than as a description of their qualities. Nevertheless, in both these cases the terms employed have the double function of naming or denoting objects, and of connoting qualities.

Mill, however, and certain other logicians who follow him, make a distinction between connotative and non-connotative terms. "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute. By a subject is here meant any-

thing which possesses attributes. Thus 'John,' or 'London,' or 'England' are names which signify a subject only. 'Whiteness,' 'length,' 'virtue,' signify an attribute only. None of these names, therefore, are connotative. But 'white,' 'long,' 'virtuous,' are connotative. The word 'white' connotes all white things, as snow, paper, the foam of the sea, etc., and implies or, as it was termed by the schoolmen, *connotes* the attribute *whiteness*. . . . All concrete general names are connotative. The word 'man,' for example, denotes Peter, James, John, and an indefinite number of other individuals, of whom, taken as a class, it is the name. But it is applied to them because they possess, and to signify that they possess, certain attributes." [1]

There is no real ground, I think, for such an absolute distinction between connotative and non-connotative terms. When we consider the use or function of terms, we find that they are never used *merely* to name things, or *merely* to connote attributes, though in certain cases the former purpose is the primary one, and in other cases the latter object is more prominent. Even when proper names are employed, the qualities or characteristics of the objects named are indirectly implied. The very fact that a proper name is given to an object implies that it possesses a certain definitely marked individuality. And a proper name when used *intelligently* carries with it some still more definite information regarding the qualities of the thing to which it is applied, as, for example, whether it is a name of a person, an animal, or a place.

[1] Mill, *System of Logic*, Bk. I. Ch. II. § 5.

The reader may consult, in connection with this chapter: —

J. S. Mill, *Logic*, Bk. I. Ch. II.
F. H. Bradley, *The Principles of Logic*, pp. 155–173.
B. Bosanquet, *Logic*, Vol. I., pp. 46–71.
" " *The Essentials of Logic*, Lecture V.

CHAPTER V

DEFINITION AND DIVISION

§ 17. **Fixing the Meaning of Terms.** — We have already referred to the necessity of definitely fixing the meaning of the terms which we employ in reasoning. In ordinary life, words are frequently used in a loose and shifting way, without any clear conception of the qualities or properties which they connote, or of the objects to which they apply. Logic demands, in the first place, that we shall have clear and definite ideas corresponding to our words, and that the signification and scope of the latter shall be carefully determined. But this is a demand to which little attention is paid in the ordinary affairs of life. To define our terms in explicit language, or even to make clear to ourselves the ideas and things for which they stand, is by no means a natural or a universal mode of procedure, but something which requires a distinct, conscious effort.

Bacon, Hobbes, Locke, Hume, and nearly all of the older philosophical writers have warned us against the abuse of words. The whole matter has been expressed very clearly by Locke, from whom I quote the following passage: —

"For he that should well consider the errors and obscurity, the mistakes and confusion, that are spread

in the world by an ill use of words will find some reason to doubt whether language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there, that when they would think on things fix their thoughts only on words, especially when they would apply their minds to moral matters; and who then can wonder if the result of such contemplations and reasonings, whilst the ideas they annex to them are very confused and very unsteady, or perhaps none at all; who can wonder, I say, that such thoughts and reasonings end in nothing but obscurity and mistake, without any clear judgment or knowledge?

"This inconvenience in an ill use of words men suffer in their own private meditations; but much more manifest are the discords which follow from it in conversation, discourse, and arguments with others. For language being the great conduit whereby men convey their discoveries, reasonings, and knowledge from one to another; he that makes an ill use of it, though he does not corrupt the fountains of knowledge which are in things themselves; yet he does, as much as in him lies, break or stop the pipes whereby it is distributed to the public use and advantage of mankind." [1]

The remedy for the obscurities and confusions of words is to be found in clear and distinct ideas. We must endeavour to go behind the words and realize clearly and distinctly in consciousness the ideas for which they stand. Now the means which logic re-

[1] *Essay concerning Human Understanding*, Bk. III. Ch. XI.

commends for the attainment of this end is definition. The first requirement of logical reasoning is that terms shall be accurately defined. There are, however, two ways in which the meaning of a term may be defined or explained. Every term, as we have already seen (§ 16), may be regarded either from the point of view of intension, or from that of extension. To define in the narrower sense is to explain from the standpoint of intension, to state the attributes or qualities which are connoted by the term. The process of explaining terms with reference to the objects, or classes of objects, for which they stand is known as Division. We may include, then, under the general term definition, (1) *Intensive definition, or definition in the narrower sense,* and (2) *Extensive definition or division.*

§ 18. **Definition.** — To define a term is to state its connotation, or to enumerate the attributes which it implies. Thus we define a parallelogram as a quadrilateral figure whose opposite sides are parallel. A distinction is often made between *verbal* and *real* definition. When we merely wish to explain the meaning in which we intend to employ some term, we have verbal definition. But when it is the purpose of our assertion to state the real nature or essential characteristics of some object, the proposition employed is said to constitute a real definition. This distinction, though not without importance, cannot, I think, be regarded as ultimate. For we never define a word or term for its own sake merely, but in order to understand the nature of the objects to which it refers. Indeed, a mere word,

apart from the things for which it stands, has no interest for us. In defining a term, then, we are always attempting to explicate or explain, more or less directly, the nature of a thing, or our idea about a thing.

Nevertheless, there is an advantage in distinguishing propositions whose *immediate* purpose is to expound the meaning of a word, from those which assert something directly of an object. the word ‘Monarchy means ~~consists~~ in the authority of one man over others,’ may be regarded as a verbal definition, because the purpose of the proposition is simply to explain the meaning of the subject term. On the other hand, ‘iron is malleable’ is a *real* definition (though not a complete one), because it does not primarily refer to the signification of the word ‘iron,’ but to the real object to which the name is applied.

In this connection, it is interesting to notice that a proposition which amounts to nothing more ~~then~~ than a verbal definition, is sometimes put forward as if it were an assertion which contained some real knowledge. The solemn commonplaces in which ignorant persons delight are often of this character. ‘A republic is a government by the people,’ ‘a just man will do what is right,’ ‘if it rains, the ground will be wet,’ may serve as examples. The mistake in such cases consists in supposing that these assertions are anything more than verbal.

There are two points of view from which the subject of definition may be considered. We might either discuss *the best method of obtaining real definitions of the nature of things*, or might confine our attention to the *requirements which a good definition has to fulfil.* A person's ability to define either a term, or the thing

for which the term stands, depends, however, upon the possession of clear and distinct ideas on the subject. The problem, then, as to the best method of finding definitions, resolves itself into an inquiry concerning the means to be used in obtaining and classifying our ideas in general; and the answer to this question, so far as an answer can be given, must be found in the theory of logic as a whole. In our treatment of the subject we shall, therefore, confine our attention mainly to a consideration of the requirements of a logical definition, and the rules which must be observed in stating it in language.

Before entering upon the subject, however, it is interesting to refer briefly to the method proposed by Socrates for obtaining definitions. Socrates, as we have already seen (§ 5), was the first to emphasize the necessity of defining and fixing the meaning of familiar terms. He found that, though the people of Athens were constantly using terms like 'good,' 'beautiful,' 'justice,' and 'temperance,' none of them, not even those with the greatest reputation for wisdom, were able to give any clear and consistent statement of what these terms implied. Socrates himself did not profess to be wiser than the rest, but he had a genuine spirit of inquiry, and made it the business of his life to try to arrive at clear conceptions, especially with regard to certain fundamental ethical virtues, like justice, and temperance, and wisdom, which he regarded as of the utmost practical importance. It was by means of conversation with others that he sought to gain clear ideas regarding the nature of these virtues. By a

series of questions and answers, by comparison of any definition proposed with particular facts which are admitted, he led his interlocutors to expose and refute the inadequacies of their earlier statements. In the *Republic*, for example, the question is regarding the nature of justice. The first definition suggested is, that it is just 'to speak the truth, and to restore to each man his own.' But supposing that a man were out of his mind and demanded his weapons which had been placed in the hands of a friend, would the friend be an unjust man if he refused to return the weapons, or abstained from telling the whole truth? Evidently not. The definition is then modified to read, 'It is just to give to each man what is his due.' Socrates then questions further, What is due to each man? What is due to a friend, and what to an enemy? This leads to the further modification that 'justice means doing good to our friends and harm to our enemies.' By referring again to particular instances and familiar analogies, Socrates leads the person maintaining this definition to admit that to injure a person is to make him less virtuous, and therefore less just. But how can justice render the character of another less just than it was before? The idea is absurd; therefore the definition has to be abandoned, and a fresh start made.

This method of proceeding by means of question and answer, and thus compelling a speaker to admit particular facts which refute the general thesis which he is maintaining, is called **Dialectic**. This was the means by which Socrates constantly strove to advance to consistent and adequate definitions. Apart from the dialectical

and dramatic form which the Socratic argument took, the method employed is essentially that of induction. For the definition, or conception, is derived from a comparison of particular instances, both positive and negative. By a consideration of individual cases, Socrates sought to obtain a definition which would be a complete and adequate expression of the nature of all the individuals which share in the class name. Aristotle says that it is to Socrates we owe the method of induction and logical definitions. Clear and distinct conceptions, formulated in exact definitions, constituted the scientific goal for Socrates, and the inductive procedure of observing and classifying particular instances was the means which he employed for reaching this goal.

The second question has reference to the formulation of a definition in language. Suppose that we already possess a clear conception of the meaning of the terms to be defined, what are the conditions which a logical definition must fulfil? The answer to this question is usually given in logical text-books by means of a set of rules for definition. Before stating these rules, however, it is necessary to explain the meaning of the terms 'genus,' 'species,' and 'differentia,' which will be frequently employed throughout the remainder of this chapter. These terms, together with 'property' and 'accident,' constitute what the older logicians call the predicables, and to which a great deal of importance was supposed to belong. It will only be necessary, however, for us to consider briefly the signification of the first three terms.

In logic, any term may be regarded as a **genus** which contains two or more subordinate classes or species. A **species**, on the other hand, is simply a subdivision or subordinate class of some larger whole. Thus 'metal' is a genus with reference to iron, gold, silver, etc., which are its species. 'Rectilinear figure' is the genus to which belong the various species, triangle, quadrilateral, pentagon, etc. The **differentia** of any term is made up of the qualities or characteristics which distinguish it from other terms, from the genus to which it belongs, as well as from the species which are coördinate with it. Thus the logical differentia of a triangle, is the property of having three sides, the differentia of man, is that which distinguishes him from other animals, whether this be the power of speech and reason, or some other characteristic either physical or mental.

The use of the terms 'genus' and 'species' in logic is entirely relative. That is, any term may be considered either as a species or a genus, according as it is regarded as forming a part of some more comprehensive class, or as itself including other classes. Thus man, for example, is a species of the genus 'animal'; but the same term also may be regarded as a genus including various species of men, Caucasians, Negroes, Mongolians, etc. In the same way, 'animal' may be considered a species of the still more comprehensive class 'organized being,' and this latter term again as a species of the genus 'material being.' A still higher or more comprehensive term which includes as its species material and spiritual beings alike is 'being.' Since this term includes every-

thing which exists, and can therefore never be included in any more 'general class, it is sometimes called the highest genus' (*summum genus*). On the other hand, we might proceed downwards until we come to a class which did not admit of division into any subordinate classes. Such a term is called in logic the lowest species (*infima species*).

It is important to notice that the terms 'genus' and 'species' have not the same signification in logic as in the natural sciences. In classifying objects in natural history, we use the terms 'variety,' 'species, 'genus,' 'family,' and 'order,' to denote varying degrees of relationship between certain groups or classes of objects. These terms, as thus employed, also indicate certain relatively fixed divisions, or permanent ways of grouping the various forms of plant and animal life. But in logic the terms 'genus' and 'species' are employed to indicate the relationship between any higher and lower class whatsoever. Moreover, as we have seen, any term (excepting only the highest genus and the lowest species) may be regarded from different standpoints, as either a genus or a species.

We shall now proceed to state the requirements of a logical definition: —

(1) *A definition should state the essential attributes of the thing to be defined.* This is done by stating the genus to which the object belongs, and also the peculiar marks or qualities by means of which it is distinguished from other members of the same class. Or as the rule is usually stated: A logical definition should give the next or proximate genus, and the differentia of the species to be defined. Thus we define a triangle as a rectilinear figure (genus), having three sides (differentia); and man as an animal (genus), which has the power of speech and reason (differentia).

(2) *A definition should not contain the name to be defined, nor any word which is directly synonymous with it.* If, for example, we were to define justice as the way of acting justly, or life as the sum of vital processes, we should be guilty of a violation of this rule.

(3) *The definition should be exactly equivalent to the class of objects defined, that is, it must be neither too broad nor too narrow.* In other words, the definition must take account of the whole class and nothing but the class. 'A sensation is an elementary state of consciousness,' for example, is too broad a definition, since it applies equally to affective and conative elementary processes. On the other hand, the definition of government as 'an institution created by the people for the protection of their lives and liberties,' is too narrow. For it takes no account of absolute forms of government which do not depend upon the will of the people. Both of these cases may be regarded as a failure to give the true differentia of the class to be defined, and hence as violations of the first rule.

(4) *A definition should not be expressed in obscure, figurative, or ambiguous language.* The reasons for this rule are at once evident. Any lack of clearness or definiteness in a definition renders it useless as an explanation. Sometimes the words used in defining may be less familiar than the term to be explained (*ignotum per ignotius*). The definition which was once given of the word 'net' as 'a reticulated texture with large interstices or meshes,' may serve as an example.

(5) *A definition should, whenever possible, be affirmative rather than negative.* A definition, that is, should

state what a term implies rather than what it does not imply. Sometimes, however, the purpose of a definition may be best attained by a negative statement of what is excluded by the meaning of the term. Thus, for example, we may define a spiritual being as a being which is not material, that is, unlike a material body made up of parts extended in space.

A logical definition, as has been said, requires us to mention the proximate genus or next higher class to which the species to be defined belongs, and also the specific or characteristic differences which distinguish it from other species. Now it is clear that there are certain cases in which these conditions cannot be fulfilled. In the first place, no logical definition can be given of the highest genus, because there is no more general class to which it can be referred. And again, although it is possible to give the differentia of any species such as 'man' or 'metal,' it is not possible to state *individual* characteristics by means of a logical definition. An individual thing may be perceived, and its various properties pointed out. But it is never possible to state in a logical definition wherein the individuality of a particular thing consists. The uniqueness of a particular object cannot be summed up in a general definition, but must be learned through perception. We may perhaps say that the highest genus is above, and the individual thing below, the sphere of logical definition.

There are, moreover, other terms such as 'space,' 'time,' 'life,' 'thought,' which are not readily referred to any higher class, and for which therefore logical definitions cannot be given. These terms are sometimes said to denote objects which are *sui generis*, or of their own class.

§ 19. **Division.** — We have already spoken of division as a process of defining a term from the point of view of extension. This is to enumerate the objects or classes of objects which the term denotes. This

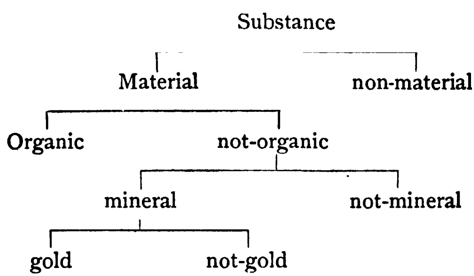
enumeration must, however, be guided by certain principles which we have now to consider.

It is usual to begin this subject by speaking of Dichotomy, or the division of a term into two parts (δίχα τέμνειν, to cut in two). This is a purely formal process, and is based on the so-called law of **Excluded Middle**, which is regarded as one of the fundamental laws of thought. This law may be stated as follows: There is no middle ground between contradictories. Any term, *a*, is either *b* or not-*b*. A triangle is either equilateral or not-equilateral. Of two contradictory predicates, one or the other must belong to every possible subject.

Now it is clear that this is a purely formal principle of division. Some positive knowledge of the particular facts involved is always necessary, in order to enable one to determine what things do stand in this relation of logical opposition. The logical law, in other words, does not help us at all in deciding what may be regarded as not-*a* in any particular case. It is not, therefore, a means of increasing our knowledge, but merely a principle of order and arrangement. This fact, obvious as it seems, was not understood by the Schoolmen who busied themselves with logic in the latter part of the Middle Ages. They clung firmly to the belief that it was possible to discover the nature of particular facts by purely formal operations of this kind. Accordingly, they spent a great deal of time in classifying and arranging terms as contradictions, contraries, etc. This work was doubtless of much service in fixing the meaning of terms, and in preventing confusion in their employment. But it was a purely verbal investigation, and of course

could not lead to any discoveries regarding the nature of things.

Moreover, it must be noticed that we do not always get propositions to which any meaning can be attached by uniting subjects and predicates in this way. If the law of Dichotomy is not guided by knowledge of the particular facts, it will give absurd propositions like, 'virtue is either square or not-square,' 'iron is either pious or not-pious.' Unmeaning propositions of this kind being left out of account, however, we may proceed to divide everything according to this principle. All geometrical figures are either rectilinear or not-rectilinear; all rectilinear figures either triangular or not-triangular; all triangles, equilateral or not-equilateral, etc. This method of division may be represented thus: —



If it were desirable, the terms 'non-material,' 'organic,' and 'not-mineral' might also be further subdivided in the same way.

Now it is not difficult to see that the practical use of this principle will depend upon our ability to find some positive value for the negative not-*a*. That is, to make the law of more than formal value, we must know what

concrete term excludes *a*, or is its logical contradictory. And knowledge of this kind comes, as already said, only from experience of the particular facts. The strictly *logical* contradictory of *a* is always not-*a*; of wise, not-wise, of cold, not-cold, etc. Mistakes frequently arise in stating contradictories in a positive form. The difficulty is that terms are chosen which are not true logical contradictories. Thus, if we say that every man is either wise or foolish, our terms are not contradictory, for a middle ground between them is possible. The same would be true of divisions like, 'large or small,' 'rich or poor,' 'saint or sinner,' 'idle or diligent.' In general, it is safe to scrutinize all dichotomic divisions very sharply to see that the alternatives are really contradictories.

The method of dichotomy depends, as we have seen, upon the law of Excluded Middle. But there is also another process called **Division** in logic, which is perhaps better known by its less technical name of Classification. In classification, there is no necessary limit to the number of classes or divisions which may be obtained. In this respect, it of course differs fundamentally from the twofold division which we have been examining. Furthermore, a classification is always made according to some principle which is retained throughout the whole process. Any common characteristic of the group of individuals to be divided may be taken as a principle of classification. If, however, the characteristic chosen is merely an external and accidental one, the classification based upon it will be regarded as *artificial*, and made for some special or temporary purposes.

Thus we might divide all flowering plants according to the color of the flowers, or the persons in any company according to the pattern of their shoes. A classification which proceeds upon such surface distinctions has, of course, no real or scientific value. It does not attempt to discover fundamental or deep-lying resemblances between the individuals with which it deals.

A scientific or natural classification, on the other hand, has for its purpose the discovery of real likeness or resemblance. It seeks to find and group together the things which are related in some essential point. Consequently, it selects as its principle of division some property which appears to be a real mark of individuality, and to be connected with changes in other properties. Such a real principle of natural classification is rarely found by comparison of merely one property or set of properties in the things to be compared. To classify according to a single property may be a convenient method of giving names to any group of individuals, and of arranging them in such a way as to be useful to the student. It does not, however, give any adequate idea of the properties and true relations of the individuals compared. A really scientific, or natural, classification must be based upon a study and comparison of all the discoverable properties of the different individuals to be classified. It is only in this way that their real resemblance and affinities can be brought to light.

(1) The classification of plants proposed by the famous Swedish botanist, Karl Linnæus (1707–1778), was based upon the comparison of a single feature: the structure of the sexual organs of plants. This method proved of the greatest convenience in indexing plants in a

convenient way into genera and species so that they could be named and described. Yet since the classification adopted was based upon a single property or feature of the plant, it was considered (even by Linnæus himself) as merely artificial. Of course it is not so obviously artificial as the examples of what we may perhaps call merely accidental or trivial classification given above. But Linnæus's system did not aim at setting forth the true relations of plants, and it was not based upon any systematic study of all their properties. It is useful merely as a stepping-stone to the real study of plants which is presupposed in natural classification.

Certain rules for division are usually given in connection with the treatment of this subject. It is not, of course, supposed that by their help one can properly divide any subject without special knowledge. The purpose of these rules is rather to warn against the logical errors to which one is most liable in the process of division.

(1) Every division is made on the ground of differences in some attribute (or attributes) common to all the members of the whole to be divided.

(2) Every division must be based on a single principle or ground (*fundamentum divisionis*).

(3) The constituent species (or groups into which the whole is divided) must not overlap, but must be mutually exclusive.

(4) The division must be exhaustive, *i.e.*, the constituent species must be equal, when added together, to the genus.

The first rule requires no remark. It simply states that it is only possible to divide any whole on the basis of differences in something which is common to all its parts. The second rule warns against changing the

principle of division while the process is being carried out. This law would be violated, if, for example, one were to divide mankind into Caucasians, Negroes, Mongolians, Europeans, Australians, and Americans. The principle of division which was first adopted in this example was obviously that of the color of the skin. But this principle was not carried through, and another principle, that of geographical distribution, was substituted for it. In dividing one must be clearly conscious of the principle which one is using, and keep a firm hold of it until the division is completed. The example which we have just given also violates the third rule. For not all of the groups, European, Caucasian, etc., exclude one another. Similarly, it would not be good logic to divide animals into vertebrates, mammals, insects, birds, molluscs, and fishes. The fourth rule simply insists that the division must be complete. The whole must be completely included in its divisions. It would not be a complete division to say that books may be divided into folios, quartos, and duodecimos; or vertebrates into mammals and birds. For in neither of these examples are the divisions enumerated equal to the whole class.

References

J. S. Mill, *Logic*, Bk. I. Chs. VII. and VIII.

W. Minto, *Logic Inductive and Deductive*, Pt. II. pp. 82–130.

C. Sigwart, *Logic*, Vol. I. §§ 42–44.

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CHAPTER VI

PROPOSITIONS

§ 20. **The Nature of a Proposition.** — A proposition is the expression in words of an act of judgment. It is composed, as we have already seen, of two terms, a subject and a predicate, connected by a copula. From the point of view of formal logic the predicate is affirmed (or denied) of the subject. When we come to consider the nature of judgment (cf. especially §§ 74, 77), we shall find reasons for questioning whether this analysis of the proposition can be taken as furnishing a correct account of what actually takes place in judgment. When we judge, we do not begin with words or terms which are not yet judgments, and then pass on to judgment by joining together the former in an external way. The conclusions which we shall have to adopt are, that terms represent ways of judging, that the simplest act of thought is already a judgment, and that thinking develops by advancing from incomplete to more complete and comprehensive judgments. The theory of the syllogism is, however, worked out on the view of the proposition already indicated. This is sufficiently accurate for practical purposes, and is not likely to lead to any serious mistakes so long as we remember that it is the proposition, rather than the actual nature of judgment, with which we are dealing.

The logical proposition, as the expression of an act of thought, corresponds to the grammatical sentence. Not every sentence, however, is a logical proposition. Sentences which express a wish or an interrogation do not *directly* enter into the process of argument at all, and may therefore be neglected for the present. The same is true of exclamatory sentences. Again, even indicative sentences frequently require to be rewritten in order to reduce them to the form of a logical proposition, which demands two terms and a copula. The sentence, 'the sun shines,' must, therefore, for purposes of logical treatment, be reduced to, 'the sun is a body which shines.' 'On the hillside deep lies the snow' is expressed as a logical proposition in some such form as this: 'The snow is a covering lying deep on the hillside.' It is very important to change the grammatical sentence to the regular form of a proposition before attempting to treat it logically.

The most general division of propositions is that which classifies them as **Categorical** and **Conditional**. A categorical proposition asserts directly, and without any condition. The predicate is either affirmed or denied unconditionally of the subject. 'A is B,' 'this room is not cold,' 'New York is the largest city in America,' are examples of categorical propositions. Conditional propositions, on the other hand, make a statement which is not immediately and directly true, but only claims to be true under a condition; as, *e.g.*, 'we shall go to-morrow, if it does not rain.' 'It will either rain or snow to-morrow,' is also a conditional proposition; for neither rain nor snow are asserted directly

and absolutely, but in each case the appearance of the one is dependent upon the non-appearance of the other.

The first of these conditional propositions is known as a *Hypothetical*, and the latter as a *Disjunctive* proposition; but for the present we shall deal only with categorical propositions, and with the form of syllogistic argument to which they give rise. After we have completed the account of the categorical syllogism, however, it will be necessary to return to a consideration of conditional propositions, and to the class of arguments in which they are employed.

§ 21. **The Quality and Quantity of Propositions.** — We shall now consider the various kinds of categorical propositions. Such propositions are classified with regard to **quality** and **quantity**. From the standpoint of quality, propositions are either **affirmative** or **negative**. An *affirmative* proposition is one in which an agreement is affirmed between the subject and predicate, or in which the predicate is asserted of the subject. The proposition, 'snow is white,' for example, indicates such an agreement between the subject and predicate, and is therefore affirmative in quality. A *negative* proposition indicates a lack of agreement or harmony between the subject and predicate. The predicate does not belong to the subject, but all relation or connection between the two is denied. 'The room is not cold,' 'the trees are not yet in full leaf,' are examples of negative propositions.

The **quantity** of a proposition is determined by the extension of the subject. When the proposition refers to all of the individuals denoted by the subject, it is said

to be universal in quantity. When, on the other hand, the proposition affirms that the predicate belongs only to a part of the subject, it is said to be particular. For example, 'all metals are elements' is a universal proposition, because the assertion is made of the subject in its widest or fullest extent; 'some metals are white' is a particular proposition, because reference is made to only a part of the subject 'metal.'

We divide propositions, then, with regard to quantity, into Universal and Particular propositions. Universal propositions are often indicated by adjectives like 'all,' 'the whole,' 'every,' etc. It frequently happens, however, that no such mark of universality is present. A scientific law is usually stated without any explicit statement of its quantity, though from its very nature it is meant to be universal. Thus we say, 'the planets revolve around the sun,' 'comets are subject to the law of gravitation.' Propositions which have a singular or an individual name as subject are ~~often called Individual~~ propositions, as, *e.g.*, 'the earth is a planet,' 'knowledge is power.' But since it is impossible to limit a singular subject, individual propositions are to be regarded as universal. They belong, that is, to the class of propositions which employ the subject term in its complete extent.

Another class, called Indefinite or Indesignate propositions, has sometimes been proposed. This class is usually said to include propositions in which the form of the words does not give any indication whether the predicate is used of the whole, or only of a part of the subject. 'Men are to be trusted,' 'animals are capable

of self-movement,' may serve as examples. This classification may be useful in illustrating the evil of making indefinite or ambiguous statements. Otherwise there is nothing to be learned from it. A really indefinite proposition has no place in an argument, and logic rightfully refuses to deal with it. The first demand of logic is that our statements shall be clear and precise. A proposition is not necessarily indefinite, however, because it has no qualifying words like 'all' or 'some.' It is the meaning of a proposition as a whole, rather than the form of its subject, which renders it definite or indefinite. Where, on the other hand, it is really impossible to decide whether the proposition is universal or particular, logic forbids us to proceed with the argument until this point has been made clear.

Particular propositions are usually preceded by some word or phrase which shows that the subject is limited in the extent of its application. The logical sign of particular propositions is 'some,' but other qualifying words and phrases, such as 'the greatest part,' 'nearly all,' 'several,' 'a small number,' etc., also indicate particularity. Here again, however, it is the meaning of the proposition, rather than its form, which is to be considered. 'All metals are not white,' for example, is a particular proposition, although introduced by 'all,' since it is clearly equivalent to 'some metals are not white.' 'Every mark of weakness is not a disgrace,' again, is a particular proposition, and signifies that 'not all, or some marks of weakness are not disgraceful.'

The words 'few' and 'a few' require special attention. The latter, as in the proposition, 'a few persons

have spoken to me about it,' is equivalent to 'some,' and introduces a particular affirmative proposition. 'Few,' on the other hand, is negative in character. Thus, 'few were saved from the shipwreck' implies that only a few were saved, or that the greater number did not escape, and the proposition is therefore to be considered as a particular negative. Propositions, then, are classified as affirmative and negative in Quality, universal and particular in Quantity. When these classifications are combined, we get four kinds of propositions, to symbolize which the vowels A, E, I, O are employed. A and I, the vowels contained in *affirmo*, stand for affirmative propositions; E and O, the vowels in *nego*, for negative propositions. This may be represented as follows: —

Universal	Affirmative:	All S is P.	A
	Negative:	No S is P.	E
Particular	Affirmative:	Some S is P.	I
	Negative:	Some S is not P.	O

We shall henceforth use A, E, I, and O to represent respectively a universal affirmative, a universal negative, a particular affirmative, and a particular negative proposition. In dealing with propositions logically, the first step is to reduce them to one or other of these four types. This can be accomplished readily by noticing the distinctions previously laid down. There are, however, certain grammatical forms and sentences which present some difficulty, and it may therefore be useful to consider them separately.

§ 22. **Difficulties in Classification.** — In the first place, we may notice that in ordinary language the terms

of a proposition are frequently inverted, or its parts separated in such a way that it requires attention to determine its true logical order. In the proposition, 'now came still evening on,' for example, the subject 'still evening' stands between two portions of the predicate. As a logical proposition, the sentence would have to be expressed in some such form as the following: 'Still evening is the time which now came on.' Similarly, we should have to write an inverted sentence like, 'deep lies the snow on the mountain,' as 'the snow is something which lies deep on the mountain.'

If a subject is qualified by a relative clause, the verb of the latter must not be confused with the main assertion of the proposition. Take the sentence, 'he is brave who conquers his passions.' Here it is evident that the relative clause describes or qualifies 'he.' Logically, then, the proposition is of the form A, and is to be written, 'he who conquers his passions is brave.' The reader will notice that all propositions which begin with pronouns like 'he who,' 'whoever,' etc., are universal in quantity, since they mean all who belong to the class in question.

(1) We have reduced grammatical sentences to logical propositions by changing the form in such a way as to have two terms united by 'is' or 'are' as the copula. Such a proposition, however, does not express time, but simply the relation existing between subject and predicate. When the grammatical sentence does involve a reference to time, and especially to past or future time, the reduction to logical form is somewhat awkward. Perhaps the best method is to throw the verb expressing time into the predicate. Thus 'the steamer will sail to-morrow' = 'the steamer is a vessel which will sail to-morrow'; 'we waited for you two hours

yesterday' = 'we are persons who waited for you two hours yesterday.'

(2) Exclusive propositions exclude all individuals or classes except those mentioned by the use of some such word as 'except,' 'none but,' 'only.' 'None but the guilty fear the judge'; 'only citizens can hold property'; 'no admittance except on business.' These propositions may all be reduced to the form E by writing 'no' before the *negative* of the subject term. Thus 'none but the guilty fear the judge' = '*no* one who is *not* guilty fears the judge'; 'only citizens can hold property' = '*no* one who is *not* a citizen, etc'; 'no admittance except on business' = '*no* person who has *not* business is to be admitted.'

§ 23. **Formal Relation of Subject and Predicate.** — We have now to consider how the relation existing between the terms of a proposition is to be understood. In § 16 it was shown that every term may be interpreted in two ways: either from the point of view of extension, or from that of intension. Extensively, terms are taken to represent objects or classes of objects; while their meaning in intension has reference to the attributes or qualities of things. Now the interpretation of the categorical proposition given by formal logic is based entirely on extension. That is, the subject and predicate are regarded as standing for individual objects or classes of objects. The question to be considered, then, concerns the extensive relation of these groups of objects in the propositions A, E, I, and O.

This mode of interpreting propositions must not be taken as furnishing an adequate theory of the nature of the act of judgment which is expressed in the proposition. It leaves entirely out of account, as we have seen, the connection of attributes asserted by the propo-

sition, which in many cases is the most prominent part of its signification. Thus the proposition, 'all metals are elements,' implies that the quality of being an element is united with the other qualities connoted by the term 'metal.' Indeed, this interpretation is perhaps more natural than the one given by formal logic, namely, that the class of metals is included in the class of elements. It must be admitted that the extensive way of reading propositions, as affirming or denying the inclusion of one class of objects in another class, frequently seems artificial. Nevertheless, it is the view upon which the historical account of the syllogism is founded. And the fact that this mode of representing the meaning of propositions leads in practice to correct conclusions, proves that it is not wholly false. It represents, as we have seen, one side or aspect of the meaning of propositions.

From the point of view of formal logic, then, a logical proposition signifies that a certain relation exists between the class of things denoted by the subject, and that denoted by the predicate. This relation may be one of inclusion or of exclusion. For example, the proposition 'all good men are charitable' is interpreted to mean that 'good men' are included in the class of 'charitable men.' On the other hand, 'no birds are mammals,' signifies that the two classes, 'birds' and 'mammals,' are mutually exclusive. The meanings of the four logical propositions A, E, I, and O may be represented by means of a series of diagrams, which were first used by the celebrated German mathematician Euler, who lived in the eighteenth century.

To represent the meaning of a proposition in A, like 'all good men are charitable,' we draw a circle to symbolize the class of charitable beings, and then place inside it a smaller circle to stand for men. The proposition, that is, signifies that 'good men' are included in the class of 'charitable beings.' The subject belongs to, or falls within, the larger class of objects represented by the predicate.

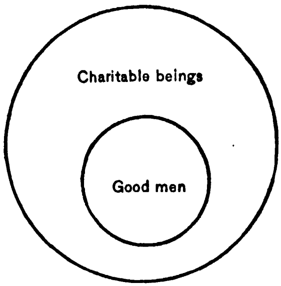

FIG. 1.

It must be carefully noted that proposition A does not usually assert anything *of the whole of its predicate.* In the example just given, no assertion is made regarding the whole class of 'charitable beings,' but only in so far as they are identical with 'good men.' There may possibly be other charitable beings who are not good men, or not men at all. The meaning of the proposition, then, is that 'all good men are *some* charitable beings.' In other words, the predicate of the ordinary universal affirmative proposition is taken only in a partial, or limited extent: nothing is affirmed of the whole of the circle of charitable beings. We denote this fact by saying that the predicate of proposition A is *undis-*

tributed. The subject, on the other hand, as a universal term, is employed in its fullest extent, or is *distributed.*

In some cases, however, the predicate is not a broader term which includes the subject, but the two are equal in extent. In the proposition, 'all equilateral triangles are equiangular,' for example, this is the case. If we were to represent this proposition graphically, the circle of equilateral triangles would not fall inside that of equilateral triangles, but would coincide with it. The same relation between subject and predicate holds in the case of logical definitions. For example, in the definition, 'monarchy is a form of political government where one man is sovereign,' the subject is coextensive with the whole of the predicate. In examples of this kind, it is of course obvious that the predicate, as well as the subject, is distributed.

As an example of proposition E, we may take the example, 'no birds are mammals.' The meaning of this proposition is represented graphically by means of two circles falling outside each other as in Fig. 2.

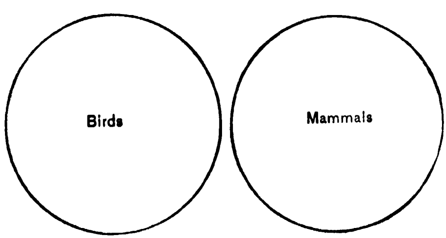

FIG. 2.

The proposition asserts that the class of birds falls completely without the class of mammals, that the two classes are entirely distinct, and mutually exclusive.

With regard to quantity, the subject is of course universal or *distributed*. And, in this case, the predicate is also distributed. For the proposition asserts that the subject 'birds' does not agree with *any part* of 'mammals.' Or, in terms of the diagram, we deny that the circle representing 'birds' corresponds with *any portion* of the circle 'mammals.' But to exclude the former circle completely from the circle which represents 'mammals,' it is necessary that we know the whole extent of the latter. Otherwise we could not be sure that the subject had not some point in common with it. Proposition E, therefore, distributes, or uses in their widest extent, both subject and predicate.

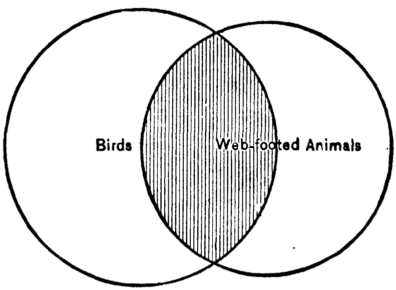

FIG. 3.

The meaning of a proposition in I, as, *e.g.*, 'some birds are web-footed,' is shown by means of two circles intersecting or overlapping as in Fig. 3. A part of the class of birds corresponds with a part of web-footed animals. The proposition has reference to the common segment of the two circles, which may be large or small. The two circles correspond in part at least. In proposition I, both subject and predicate are undistributed. The

subject is, of course, a particular or limited term. And, as will be clear from what has already been said in the case of proposition A, reference is made to only a limited portion of the predicate. In the example used, the assertion refers only to those web-footed animals which are also birds. Or we may say that the proposition has reference only to the common segment of the circles representing subject and predicate. Nothing is asserted of the other portions of the two circles. In other words, both subject and predicate are employed in a limited extent, or are *undistributed*.

'Some metals are not white,' may serve as an example of proposition O.

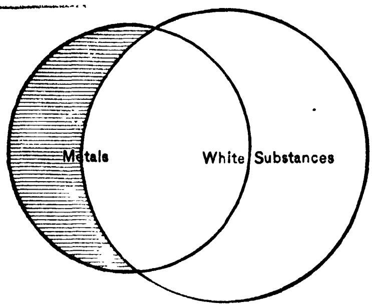

FIG. 4.

This proposition may be represented graphically as in Fig. 4. Though this is the same form of diagram as that employed in the last figure, the proposition refers now to the outlying part of the circle 'metals.' Some metals, it asserts, do not fall within the sphere of white substances. A larger or smaller section of the circle representing the former term, falls *completely without* the circle of white substances.

It is necessary to notice carefully that although the subject of O is undistributed, its predicate is distributed. For, as we have seen, a part of the subject is completely excluded from the class of 'white substances.' But in order to exclude from *every part* of the predicate, the full extent of the predicate must be known. Or, in terms of the diagram, the proposition excludes a portion of the circle of metals (some metals) from each and every part of the circle of white things. The latter term must therefore be used in its full extent, or be distributed.

It is absolutely necessary, in order to comprehend what follows, to understand the distribution of terms in the various propositions. It may help the reader to remember this if we summarize our results in the following way: —

Proposition A, subject distributed, predicate undistributed.
Proposition E, subject distributed, predicate distributed.
Proposition I, subject undistributed, predicate undistributed
Proposition O, subject undistributed, predicate distributed.

References to § 23

W. S. Jevons, *Elementary Lessons in Logic*, pp. 71–75.
J. S. Mill, *Logic*, Bk. I. Ch. V.
C. Sigwart, *Logic*, § 5.
B. Bosanquet, *The Essentials of Logic*, Lectures V. and VI.

CHAPTER VII

THE INTERPRETATION OF PROPOSITIONS

§ 24. **The So-called Process of Immediate Inference.**— Many logicians speak of two kinds, or processes of reasoning, to which they give the names of **mediate**, and **immediate inference.** Mediate inference, it is said, asserts the agreement or disagreement of a subject and predicate after having compared each with some common element or middle term. The conclusion is thus reached mediately or indirectly. The syllogism is the best example of mediate inference. In the syllogism,

All M is P,
All S is M,
Therefore S is P,

the conclusion is reached through the medium of M, with which both S and P have been compared. It will be noticed that to obtain a conclusion in this way two propositions or premises are necessary.

We sometimes are able, however, to pass directly or immediately from one proposition to another. For example, the proposition that 'no men are infallible,' warrants the statement that 'no infallible beings are men.' Or, if we know that it is true that 'some birds are web-footed,' we perceive at once that the proposition, 'no birds are web-footed,' is false. It is this process of passing directly from one proposition to another which has been named by many logicians **immediate inference.**

Can we be properly said to infer at all when we pass from one proposition to another, as in the above examples? As we have already shown, inference is a process of exhibiting the relation of facts to one another by discovering some common element, or connecting principle by means of which they are united (cf. also § 87). Wherever we can discover a connecting thread, or common element between two facts or groups of facts, we are able to *infer* with greater or less certainty from the nature of the one what the nature of the other must be. But it is essential to inference that there shall be a real transition from one fact to another — that the conclusion reached shall be different from the starting-point.

The point at issue, therefore, is whether a new fact or truth is reached in the so-called processes of immediate inferences, or whether we have the same fact repeated in the form of a new proposition. When we pass from 'no men are infallible,' to 'no infallible beings are men,' can we be said to infer a new truth? In this case it is evident, I think, that there has been no real development or extension of the original proposition so as to include a new fact. The new proposition is the result of a verbal interpretation of the original one, and restates the same fact in a different way. Inference always completes or enlarges the truth from which it sets out by showing the reasons which support it, or the consequences which follow from it. But when we pass directly from one proposition to another, as in the examples given above, it will be found, I believe, that nothing has really been added to the original statement—no new facts have been brought into connection in the process.

It is of course true that the claims of each of the different types of so-called immediate inference should be examined separately. But it will be found, I think, that the conclusion which we have reached is equally true of all of the forms to which this name is applied. It seems better to regard these processes as acts of verbal interpretation, or explication of the meaning of propositions, rather than as inferences in the true sense of the word. They render important service in helping us to understand what is implied or involved in the propositions we use, but they do not lead the mind on to any new truth. We may consider three ways in which propositions may be transformed as a result of the interpretative process — Opposition, Obversion, and Conversion.

§ 25. **The Opposition of Propositions.** — We have seen that all categorical propositions have to be reduced to one of the four forms, A, E, I, O, in order to be dealt with by logic. Now, when these propositions have the same subject and predicate, certain relations exist between them, to which the general name of Opposition has been given. It is clear that the truth of some of these propositions interferes with the truth of others. Thus if it be true that 'no professional gamblers are honest,' it is impossible that 'all professional gamblers are honest,' or even that some are honest. The proposition E is thus inconsistent with both A and I. Again, if it be false that 'all politicians are dishonest,' it must be true that 'some politicians are not dishonest,' though it by no means follows that 'no politicians are dishonest.'

That is, when A is false, O is necessarily true, while E may or may not be true. Propositions A and E are called **contrary** propositions. 'All A is B,' and 'no A is B,' express the greatest possible degree of contrariety or opposition. If one proposition be true, the other is necessarily false. It is to be noticed, however, that we cannot conclude that if one is false, the other is true. For both A and E may be false. Thus, for example, the propositions, 'all men are wise,' and 'no men are wise,' are both false. But, on the other hand, propositions A and O, E and I, are pairs of **contradictory** propositions: if one is false, its contradictory is necessarily true; and if one is true, the other is manifestly false.

The relation of the four logical propositions is clearly shown by arranging them in the following way:—

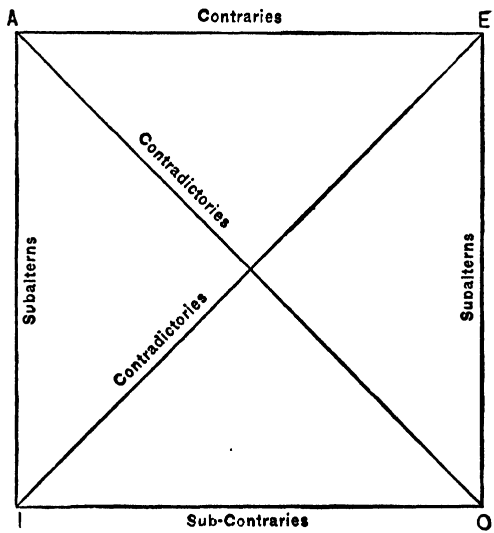

FIG. 5.

A and E are known as *contraries;* I and O as *subcontraries;* A and O, I and E, as *contradictories;* A and I, E and O, are *subalterns.*

The relations of these propositions may now be summed up in the following statements: —

(1) Of contrary propositions, one is false if the other is true, but both may be false. may be [illegible]

(2) Of contradictory propositions, one is true and the other necessarily false.

(3) If a universal proposition is true, the particular which stands under it is also true; but if the universal is false, the particular may or may not be true.

(4) If a particular proposition is true, the corresponding universal may or may not be true; but if the particular is false, the universal must be false.

(5) Subcontrary propositions may both be true; but if one is false, the other is necessarily true.

The knowledge that any one of these propositions is either true or false enables us to determine the truth or falsity of at least some of the others.

For example, if A is true, E is false, O is false, and I is true. If A is false, E is doubtful, O is true, and I doubtful.

If I is true, E is false, A is doubtful, and O doubtful. If I is false, E is true, A is false, and O true.

Similarly we are also able to determine what follows when we suppose that E and O are either false or true.

It ought to be carefully noted that when we affirm the truth of the particular proposition I, we do not deny the truth of the universal proposition A. The proposition, 'some students are fond of recreation,' for example, does not exclude the truth of 'all students are

fond of recreation.' Similarly, the truth of O does not exclude the corresponding proposition in E: the statement, 'some men are not generous,' for example, does not interfere with the truth of the universal proposition, 'no men are generous.' A particular proposition, in other words, asserts something of a limited part of a subject; it neither affirms nor denies anything of the same term taken universally.

The reader will remember that propositions which have the name of some singular or individual thing as subject, have been classified as universal. 'New York is the largest city in America,' 'charity is not the only virtue,' are examples of such propositions. Now it is at once evident that in cases of this kind there are no corresponding particular propositions. What has just been said regarding the relation of universal and particular propositions, applies therefore only to propositions which have a general term or name as subject. Moreover, we must notice that when A and E propositions have a singular or individual name as subject, the relations between them are somewhat different from those just stated. A and E, we said, are contrary, but not contradictory propositions. By that it was implied that although we can proceed from the truth of the one to the falsity of the other, it is not possible to go in a converse direction, from falsity to truth. We cannot conclude, for example, from the falsity of the proposition that 'all men are selfish' the truth of the corresponding negative proposition, 'no men are selfish.' With contradictory propositions, however, we can go from a denial to an affirmation. Now the point to be observed, with regard to propositions with a singular term as subject,

is that although only contraries in form, they have yet the force of contradictories. 'Socrates is wise' (A), and 'Socrates is not wise' (E), are contradictory as well as contrary, propositions.

§ 26. **The Obversion of Propositions.** — The terms 'Obversion' and 'Æquipollence' were formerly used to denote any process by which the form of a proposition is changed without an alteration in meaning being involved. The name 'Obversion' is, however, now generally employed to describe the change which a proposition undergoes in passing from the affirmative to the negative, or from the negative to the affirmative form while still retaining its original meaning.

Every fact is capable of expression either in the form of an affirmative or of a negative proposition. Whether the affirmative or negative form is chosen in any particular case, is partly a matter of convenience. It is also determined largely by the psychological interest of the moment, *i.e.*, by the purpose which we have in view in making the assertion. When, for example, we wish to repel some suggestion which may have occurred to us, or to deny something which our companions appear to believe, we naturally choose the negative form of statement. But the meaning of the proposition is the same whether we say, 'all men are fallible,' or, 'no men are infallible.' Similarly, we can say, 'not one of the crew escaped,' or, 'all of the crew perished.'

Obversion, then, is the process of substituting for any affirmative proposition its equivalent in negative form, or of expressing the meaning of a negative prop-

osition as an affirmative. To obtain the obverse of proposition A, we proceed on the principle that two negatives are equal to an affirmative. Instead of 'all animals digest food,' we may write, 'no animals are beings that do not digest food'; for, 'every man has his own troubles,' 'there are no men who have not their own troubles.' Instead of affirming the predicate of the subject, the obverse of A takes the negative of the original predicate and denies it universally.

Proposition I may be obverted in the same way, though it yields a particular, instead of a universal negative proposition. Thus the obverse of, 'some of the houses are comfortable,' is 'some of the houses are not not-comfortable, *i.e.* uncomfortable. We deny the negative predicate in the obverse proposition, instead of affirming the positive.

We obtain the obverse of the propositions E and O by changing the negation contained in them to its equivalent affirmation. This is done by attaching the negative to the predicate, and then affirming it of the subject. For example, to obtain the obverse of, 'no one who was present can forget the scene,' we first write the proposition in logical form, 'no one who was present is a person who can forget the scene.' Now the negative of the predicate term, 'a person who can forget the scene,' is, 'a person who can *not* forget the scene.' Affirming this universally we get, 'all persons who were present are persons who cannot forget the scene.' As an example of how the obverse of O is obtained, we may take the proposition, 'some metals are not white.' Now if we change the quality of the proposition by attaching the

negative to the predicate, we obtain 'some metals are not-white.' That is, instead of denying, we affirm the negative of the original predicate. When the predicate is made up of several words, it is important that the logical contradictory of the whole term be taken. For example, in the proposition, 'some men are not fond of work,' the predicate fully expressed is, 'persons who are fond of work.' Now the negative or contradictory term corresponding to this is, 'persons who are *not* fond of work.' The obverse of the original proposition therefore is, 'some men are persons who are not fond of work.'

§ 27. **The Conversion of Propositions.** — To convert a proposition is to transpose its subject and predicate so that each shall occupy the place previously held by the other. Thus the proposition, 'no men are infallible,' is converted by writing it, 'no infallible beings are men.' The original proposition is called the **convertend**, and the proposition obtained by conversion the **converse.** By conversion, then, a new proposition is derived directly from an old one. It is for this reason that conversion is usually ranked as a process of immediate inference. But, as we have already seen, the process of interpretation which results in conversion seems to fall wholly within the proposition. In other words, it makes clear what is involved in the original proposition, but does not lead to any new fact with which the latter is connected. We therefore reached the conclusion that it might more properly be regarded as a process of formal interpretation, than as one which involves real inference.

It is evident that in proceeding to convert propositions

it will be necessary to notice whether the predicate of the convertend, or proposition to be converted, is distributed or undistributed, otherwise we should not know what extension to apply to this term when used as the subject of the converse proposition. The rules usually given to limit the process of conversion are as follows: —

(1) No term must be distributed in the converse proposition which was not distributed in the convertend.

(2) The quality of the converse proposition must remain the same as the quality of the convertend.

The reason for the first rule is at once evident from what has been already said. The second rule is not one which is always observed. Of course, the meaning of a proposition must not be altered by changing the quality simply or directly. But, in converting by Contraposition, as we shall see later, it is first necessary to obtain the equivalent of the convertend by obversion, and this necessarily involves a change of quality.

There are three kinds of conversion usually recognized: (*a*) Simple Conversion; (*b*) Conversion by Limitation or *per accidens*; (*c*) Conversion by Contraposition.

(*a*) By Simple Conversion is meant the direct transposition of the subject and predicate without any other change in the form of the proposition. Both propositions E and I can be converted in this way. Thus the converse of, 'none of the books on this shelf are novels,' is another proposition in E, 'no novels are books on this shelf.' From 'some dicotyledons are exogens' we obtain by conversion another particular affirmative proposition, 'some exogens are dicotyledons.'

(*b*) Conversion by Limitation or *per accidens* is applied to proposition A. In this process A loses its universality, and yields as a result only proposition I. To illustrate this mode of conversion we may take the proposition, 'brown hematite is an iron ore.' As we already know, the term 'an iron ore,' being the predicate of proposition A, is undistributed. When used as the subject of a new proposition, therefore, it must be limited by the adjective 'some.' We thus obtain the converse proposition, 'some iron ore is brown hematite.' Similarly, the converse of the proposition, 'all sensations are mental processes,' is 'some mental processes are sensations.' When proposition A is converted by limitation, then, it yields proposition I as a result. And it is evident that the proposition has really lost something in the process. For it is impossible by converting again to obtain anything more than a particular proposition. It is, however, sometimes possible to convert proposition A without limiting the predicate. In formal definitions, for example, the subject and the predicate are of equal extent, and may be transposed simply without any limitation of the latter. Thus the converse of, 'an equilateral triangle is a plane figure having three equal sides,' is 'a plane figure having three equal sides is an equilateral triangle.'

(*c*) In Conversion by Contraposition the negative or contradictory of the original predicate is taken as the subject of the converse proposition. This method of conversion is usually applied only to propositions A and O.

When applied to A, it means that from a proposition

in the form, All B is C, we are able to assert something of what is not C. If we know, for example, that 'all the planets are bodies revolving around the sun,' we can obtain by contraposition the proposition, 'no bodies which do not revolve around the sun are planets.' The rule for contraposition is, *first obvert, and then convert simply.* Thus, the obverse of, 'aluminium is a white metal,' is the proposition in E, 'aluminium is not a metal which is not white;' and converting this simply, we get as the contrapositive of the proposition from which we started, 'no metal which is not white is aluminium.'

Proposition O can be converted only by contraposition. If we were to convert simply, as, *e.g.*, 'some metals are not white,' 'some white things are not metals,' we should fall into error; for the term 'metal' is distributed in the converse proposition without having been distributed in the convertend. — Illicit —

To obtain the converse of O by contraposition, the rule given above, first obvert and then convert simply, applies once more. The obverse of the proposition in O, 'some men who make loud professions are not to be trusted,' is the equivalent in I, 'some men who make loud professions are persons not to be trusted.' Converting this simply, we obtain the contrapositive, 'some persons not to be trusted are men who make loud professions.'

For the sake of convenience we may sum up the treatment of Conversion as follows: —

Proposition A is converted (1) *by Limitation, and* (2) *by Contraposition.*

All S is P. (A)

(1) Converting by Limitation, Some P is S. (I)

(2) Converting by Contraposition { i.) Obversion yields, No S is not-P. (E)
ii.) The Simple Converse of this is, No not-P is S. (E)

Proposition I is converted Simply.

Some S is P. (I)

Converting Simply, Some P is S. (I)

Proposition E is converted Simply.

No S is P. (E)

Converting Simply, No P is S. (E)

Proposition E may also be converted by Contraposition, but the result is the same as the Contrapositive of O. Thus for example:—

No S is P. (E)

Converting by Contraposition { i.) Obversion yields, All S is not-P. (A)
ii.) Converting this by Limitation, Some not-P is S. (I)

Proposition O is converted by Contraposition.

Some S is not P. (O)

Converting by Contraposition { i.) Obversion yields, Some S is not-P. (I)
ii.) The Simple Converse of this is, Some not-P is S. (I)

References

B. Bosanquet, *Logic*, Vol. I. pp. 310–319.

W. Minto, *Logic Inductive and Deductive*, Pt. III. pp. 130–166.

J. H. Hyslop, *The Elements of Logic*, Ch. X.

CHAPTER VIII

THE SYLLOGISM

§ 28. **The Nature of Syllogistic Reasoning.** — The syllogism, as we have already seen (§ 10), presents a conclusion together with the reasons by means of which it is supported. A single proposition taken by itself is dogmatic: it merely asserts without stating the grounds upon which it rests. The syllogism, on the other hand, justifies its conclusion by showing the premises from which it has been derived. It thus appeals to the reason of all men, and compels their assent. To do this, it is of course necessary that the truth of the premises to which appeal is made should be granted. If the premises are disputed or doubtful, the argument is pushed a step further back, and it is first necessary to show the grounds upon which these premises rest. The assumption of syllogistic reasoning — and, indeed, of all reasoning whatsoever — is that it is possible to reach propositions which every one will accept. There are certain facts, we say, well known and established, and these can always be appealed to in support of our conclusions. In syllogistic reasoning, then, we exhibit the interdependence of propositions; *i.e.*, we show how the truth of some new proposition, or some proposition not regarded as beyond question, follows necessarily

from other propositions whose truth every one will admit.

The question which arises in connection with the syllogism, therefore, is this: Under what conditions do propositions which are accepted as true contain or imply a new proposition as a conclusion? Or we may put the question in this form: In what ways may the four logical propositions, A, E, I, O, be combined so as to yield valid conclusions?

We pointed out in a previous chapter that a syllogism has always two premises. It is, however, impossible to obtain a conclusion by combining any two propositions at random, as *e.g.*, —

All A is B.
No X is Y.

It is evident that *any* two propositions will not yield a conclusion by being taken together. In order to serve as premises for a syllogism, propositions must fulfil certain conditions, and stand in certain definite relations to each other. To determine some of the most apparent of these conditions, let us examine the argument: —

All mammals are vertebrates,
The whale is a mammal,
Therefore the whale is a vertebrate.

It will be noticed that the term 'mammal' is common to both premises, and that it does not occur at all in the conclusion. Moreover, it is because the other terms are compared in turn with this common or **Middle Term** and found to agree with it, that they can be united in the conclusion. It ~~is only propositions~~ which have a middle term, therefore, which can be employed as the

premises of a syllogism. The syllogism is thus essentially a process of comparison. Each of the terms entering into the conclusion is compared in turn with the same middle term, and in this way their relation to each other is determined. We reach the conclusion not directly or immediately, but by means of the middle term. The conclusion is therefore said to be *mediated*, and the process itself is sometimes called **mediate reasoning**.

It will be interesting to compare what has just been said regarding the function of the middle term, with what has been previously stated regarding the nature of inference. When we infer one fact from another, it was said, we do so by discovering some identical link or connecting thread which unites both. We may say that to infer is to see that, in virtue of some identical link which our thought has brought to light, the two facts, or groups of facts, are in a certain sense identical. Now the middle term in a syllogism is just the explicit statement of the nature of this identical link. It is true that in the syllogism we seem to be operating with words or terms rather than with the thought-process itself. When we go behind the external connection of the terms, however, we can see that the middle term represents the universal principle, by means of which the conclusion is reached. In the example given above, for instance, we reason that the whale, *being a mammal*, is a vertebrate.

The terms which enter into the conclusion of a syllogism are sometimes called the *Extremes*, as opposed to the middle term. Of the Extremes, *the predicate of the conclusion is known as the* **Major Term**, *and the subject of the conclusion as the* **Minor Term**. The premise which contains the major term is called the **Major Premise**, and stands first when the syllogism is arranged in logical form. The **Minor Premise**, on the other hand, is the

premise which contains the minor term, and stands second in the arrangement of the syllogism. The propositions of which the syllogism is composed may occur, however, in any order in actual reasoning; — either premise, or even the conclusion, may stand first. To arrange an argument, therefore, it is necessary to determine which is the major, and which the minor premise. This can be done only by turning to the conclusion, and distinguishing the major and minor terms. For example, take the syllogism: —

> The whale suckles its young,
> No fish suckles its young,
> Therefore the whale is not a fish.

By turning to the conclusion we see that 'fish' (being the predicate) is the major term. The proposition which contains this term, 'no fish suckles its young,' is, therefore, the major premise, and should stand first. Before proceeding to examine the syllogism further it would be necessary to arrange it as follows: —

> No fish is an animal which suckles its young,
> The whale is an animal which suckles its young,
> Therefore the whale is not a fish.

§ 29. **The Rules of the Syllogism.** — It is customary to give a number of rules or canons to which the syllogism must conform in order to yield valid conclusions. We shall first enumerate the rules, and afterwards remark on their meaning and importance.

(1) In every syllogism there should be three, and only three, terms, and these terms must be used throughout in the same sense.

The terms, as we have already remarked, are known as the major term, the middle term, and the minor term.

(2) Every syllogism contains three, and only three, propositions.

These are called the major premise, minor premise, and conclusion.

(3) The middle term must be distributed in at least one of the premises.

(4) No term must be distributed in the conclusion which was not distributed in one of the premises.

(5) From negative premises nothing can be inferred.

(6) If one premise be negative, the conclusion must be negative; and, conversely, to prove a negative conclusion one of the premises must be negative.

As a consequence of the above rules there result two additional canons which may be set down here.

(7) No conclusion can be drawn from two particular premises.

(8) If one of the premises be particular, the conclusion must be particular.

The reason for the first and second rules will be evident from what has been already said about the structure of the syllogism. We saw that a logical argument is a process of comparison; that two terms are united through comparing them with a common or middle term. If the meaning of the terms does not remain fixed, there are more than three terms, and no comparison is possible. The second rule follows as a corollary from the first.

The third rule, that the middle term must be distributed once, at least, is extremely important, and its

necessity will be readily perceived. For, since the middle term is the standard of comparison, it must be used in at least one premise in its universal extent. Otherwise we might compare the major term with one part of it, and the minor term with another part. Such a comparison would of course not warrant us in either affirming or denying the connection of these terms in the conclusion. For example, the two propositions,

> Sedimentary rocks are stratified substances,
> Some metamorphic rocks are stratified substances,

do not distribute the middle term, 'stratified substances,' at all, being both affirmative propositions. It

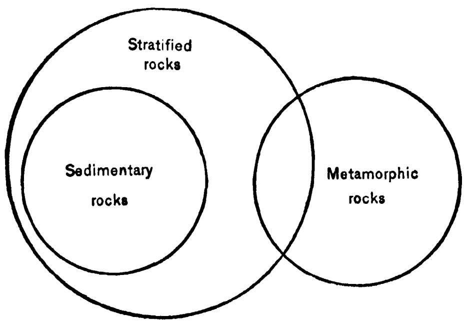

FIG. 6.

is clear that the term, 'sedimentary rocks,' agrees with one part of the stratified substances, and 'metamorphic rocks' with another part. We are, therefore, not able to infer that 'some metamorphic rocks are sedimentary rocks.' This may be clearly shown by representing the propositions by Euler's method of circles as in Fig. 6. We know from the second proposition that the circle representing 'metamorphic rocks' falls partly within the

circle of 'stratified substances.' But it is impossible to determine from the statement whether it corresponds at all with the circle of sedimentary rocks, or falls, as in the figure, entirely without it.

The fourth rule states that no term must be distributed in the conclusion which was not distributed in one of the premises. That is, the conclusion must be proved *by means of* the premises, and no term which was not employed in its universal signification in the premises can, therefore, be used universally or distributively in the conclusion. This rule may be violated by using either the major or the minor term in a wider sense in the conclusion than in the premise in which it occurs. The resulting fallacies are then known as the **Illicit Process** of the major and minor terms respectively. As an illustration of the illicit process of the major term, we may consider the following argument:—

> All rational beings are responsible for their actions,
> Brutes are not rational beings,
>
> Therefore brutes are not responsible for their actions.

It will be at once seen that the major term, 'beings responsible for their actions,' is distributed in the conclusion, but was not distributed when it appeared as the predicate of an affirmative proposition in the major premise. The fallacious nature of this argument may also be shown by representing the proposition by circles.

The illicit process of the minor term is usually more easily detected. We may take as an example of this fallacy:—

All good citizens are ready to defend their country,
All good citizens are persons who vote regularly at elections,

---

Therefore all who vote regularly at elections are ready to defend their country.

It is clear that the minor term, 'persons who vote regularly at elections,' is undistributed when used as the predicate of the minor premise. In the conclusion, however, it is wrongly taken universally, and it is this unwarranted extension to which the name of *illicit minor* is given. Students are advised to draw circles to illustrate the nature of this fallacy.

The fifth and sixth rules have reference to negative premises. It is not difficult to understand why two negative premises cannot yield any conclusion. For, from the fact that S and P are both excluded from M, we can conclude nothing regarding their relation to each other. Two negative premises afford us no standard by means of which we can determine anything concerning the relation of major and minor terms. Again, where one premise is negative and the other affirmative, it is asserted that, of the major and minor terms, one agrees, and the other does not agree, with the middle term. The necessary inference from these premises, then, is that major and minor terms do not agree with each other. That is, the conclusion must be negative.

It is worth noticing that it is sometimes possible to obtain a conclusion from premises which are both negative in form. For example: —

No one who is not thoroughly upright is to be trusted,
This man is not thoroughly upright,

Therefore this man is not to be trusted

In this example, although the form of both premises is negative, the minor premise supplies a positive basis for argument, and is really affirmative in character. Or we may say that the 'not' in the predicate of the minor premise belongs to the predicate, and not to the copula. The proposition may therefore be said to affirm, rather than to deny.

The seventh and eighth rules, which refer to particular premises, can be proved by considering separately all the possible cases. If this is done, it will be found that these rules are direct corollaries from the third and fourth, which are concerned with the proper distribution of terms. It is impossible to secure the necessary distribution with two particular premises; for either the distribution of the middle term will not be provided for, or if this has been secured by means of a negative premise, the conclusion will show a case of the illicit major term. By means of the same rules, it may be shown that a particular premise always requires a particular conclusion. The truth of these two subordinate canons may be also readily shown by the use of circles.

§ 30. **The Figures of the Syllogism.** — We have seen what an important part the middle term plays in the syllogism. It constitutes the mediating link between the major and minor terms, and makes possible their union. Now upon the position of the middle term in the premises depends the **Figure** of the syllogism. There are four possible arrangements of the middle term in the two premises, and therefore four figures of the syllogism. If we let P represent the major term, S the minor, and M the middle term, the form of the different figures may be represented as follows: —

FIRST FIGURE	SECOND FIGURE
M — P	P — M
S — M	S — M
∴ S — P	∴ S — P

THIRD FIGURE	FOURTH FIGURE
M — P	P — M
M — S	M — S
∴ S — P	∴ S — P

In the first figure, the middle term is the subject of the major premise, and the predicate of the minor premise.

In the second figure, the middle term is predicate of both major and minor premises.

The third figure has the middle term as the subject of both premises.

In the fourth figure, the middle term occupies just the opposite position in the two premises from that which it held in the first figure; *i.e.*, it is the predicate of the major premise, and the subject of the minor premise.

CHAPTER IX

THE VALID MOODS AND THE REDUCTION OF FIGURES

§ 31. **The Moods of the Syllogism.** — By the **Mood** of a syllogism we mean the combination of propositions A, E, I, and O, which goes to make it up. Thus, when a syllogism is made up of three universal affirmative propositions, we speak of it as the mood AAA; if it is composed of a universal negative, a particular affirmative, and a particular negative proposition, we name it the mood EIO.

Every syllogism, as has been already stated, is made up of some arrangement of the four propositions A, E, I, O, taken three at a time. Now, there are in all sixty-four possible permutations of these four propositions taken three at a time. We might then write out these sixty-four moods, and proceed to determine which of them are valid. But this would be a long and somewhat tedious undertaking. Moreover, if we can determine what are the valid premises, we can draw the proper conclusions for ourselves. Since, then, there are but two premises in each syllogism, we shall have to deal only with the possible permutations of A, E, I, and O, taken two at a time, or with sixteen combinations in all.

The following, then, are the only possible ways in which the propositions A, E, I, and O can be arranged as premises: —

16 cont:	16 cont.	16 cont.	16 cont.
AA	EA	IA	OA
AE	EE	IE	OE
AI	EI	II	OI
AO	EO	IO	OO

Some of these premises, however, cannot yield conclusions, since they plainly violate certain rules of the syllogism. The combinations of negative premises EE, EO, OE, and OO can be at once struck out. Again, since no conclusion follows from two particular premises, we can eliminate II, IO, and OI. There remain, then, for further consideration the combinations:—

AA	EA	IA	OA
AE	—	IE	—
AI	EI	—	—
AO	—	—	—

At this point we must recall the fact that every argument must belong to one of the four figures. We must now therefore ask this question: Which of the above combinations of premises will yield valid conclusions in the first, second, third, and fourth figures, respectively? By examining the form of the syllogism in each of these figures, we shall be able to discover what conditions must be fulfilled in each case, and to lay down special canons for each figure. We shall first proceed to state and prove the special canons of the different figures. It will not, however, be necessary for the student to commit these rules to memory, as he can always derive them for himself by a consideration of the form of the argument in the different figures.

§ 32. **The Special Canons of the Four Figures.** — *In the first figure, the minor premise must be affirmative, and the major premise universal.*

The first figure is of the form: —

$$\begin{array}{l} \quad M — P \\ \quad S — M \\ \hline \therefore S — P \end{array}$$

To show that the minor premise is affirmative, we employ the indirect method of proof. Let us suppose that the minor premise is not affirmative, but negative. Then since one premise is negative, the conclusion must be negative. But if the conclusion is a negative proposition, its predicate, P, must be distributed. Any term which is distributed in the conclusion must, however, have been distributed when it was used in the premise. P must be distributed, therefore, as the predicate of the major premise. But since negative propositions alone distribute their predicates, the major premise, M — P, must be negative. But by hypothesis the minor premise, S — M, is negative. We have, therefore, two negative premises, which is impossible. Our supposition, that the minor premise is negative, is therefore false; or, in other words, the minor premise must be affirmative.

This having been established, we can very easily prove that the major premise must be universal. For the middle term, M, must be distributed in at least one of the premises. But it is not distributed in the minor premise, for it is there the predicate of an affirmative proposition. It must, therefore, be distributed as the

subject of the major premise, that is, the major premise must be universal.

If we turn now to the second figure, we shall find that the following rules may be deduced from a consideration of its form:—

(1) *One premise must be negative, and the conclusion therefore negative.*

(2) *The major premise must be universal.*

The second figure is in the form:—

$$\begin{array}{rc} & P - M \\ & S - M \\ \hline \therefore & S - P \end{array}$$

The reason for the first rule is at once evident. If one premise is not negative, the middle term, M, is not distributed, and no conclusion is therefore possible. The only means of securing distribution of the middle term in the second figure is by means of a negative premise. And if one premise is negative, it of course follows that the conclusion must be negative.

This having been established, the proof of rule 2 follows almost immediately. For, since the conclusion is negative, its predicate, P, must be distributed. And since P is distributed in the conclusion, it must have been used distributively when it occurred as the subject of the major premise, or, in other words, the major premise must be universal.

The third figure is of the form:—

$$\begin{array}{rc} & M - P \\ & M - S \\ \hline \therefore & S - P \end{array}$$

From an analysis of this, the two following rules may be obtained: —

(1) *The minor premise must be affirmative.*

(2) *The conclusion must be particular.*

The minor premise is here shown to be affirmative by the method employed in proving the same rule in the first figure. That is, we suppose the minor premise negative, and show that, as a result of this hypothesis, the conclusion is negative, and the major term distributed. It follows, then, that this term must be distributed as the predicate of the major premise. But this could happen only if this premise were negative. The hypothesis that the minor premise is negative thus leads to the absurdity of two negative premises. The conclusion that the opposite is true, that the minor premise is affirmative, is therefore proved indirectly.

Since the minor premise is affirmative, its predicate S is undistributed. This term must therefore be used in an undistributed, *i.e.*, particular sense in the conclusion. And, as this term forms its subject, the conclusion is particular.

In the fourth figure the terms are arranged in the following way: —

$$\begin{array}{c} P - M \\ M - S \\ \hline \therefore S - P \end{array}$$

From a consideration of the form of this figure we can obtain the following special canons: —

(1) *If either premise be negative, the major premise must be universal.*

(2) *If the major premise be affirmative, the minor must be universal.*

(3) *If the minor premise be affirmative, the conclusion must be particular.*

The student will be able to prove these canons for himself by applying the rules of the syllogism in the same way as has been done in the proofs already given.

§ 33. **The Determination of the Valid Moods in Each of the Figures.** — We have now to apply these special canons in order to determine what moods are valid in each of the four figures. It has already been shown (p. 116) that the premises which are not excluded by the general rules of the syllogism are: —

AA	EA	IA	OA
AE	—	(IE)	—
AI	EI	—	—
AO	—	—	—

Now we have proved that in the first figure the major premise must be universal, and the minor affirmative. The only combinations of premises which will stand these tests are, AA, EA, AI, and EI. Drawing the proper conclusion in each case, we have as the four valid moods of the first figure: —

AAA, EAE, AII, EIO.

It will be noticed that the first figure enables us to obtain as conclusion any one of the four logical propositions, A, E, I, and O.

The special canons of the second figure state that

the major premise must be universal, and one premise negative. Selecting the combinations of premises which fulfil these conditions, we obtain EA, AE, EI, and AO. These give, when the conclusions have been drawn, the following four moods of the second figure: —

EAE, AEE, EIO, AOO.

By means of the second figure, therefore, we are able to establish the truth only of the negative propositions, E and O.

In the third figure the minor premise must be affirmative, and the conclusion particular. Taking all the combinations in which the minor is affirmative, there result, AA, IA, AI, EA, OA, EI. It must be remembered that the third figure yields only particular conclusions, even where both premises are universal. The valid moods in this figure are therefore as follows: -

AAI, IAI, AII, EAO, OAO, EIO.

The canons of the fourth figure, which have to do with the premises, state that where either premise is negative, a universal major is necessary, and that an affirmative major premise must be accompanied by a universal minor. The combinations of propositions which fulfil these conditions are AA, AE, IA, EA, and EI. In drawing conclusions from these premises, however, it is necessary to pay attention to the third canon of this figure, which states that where the minor premise is affirmative, the conclusion must be particular. Accordingly, the valid moods of this figure may now be written: —

AAI, AEE, IAI, EAO, EIO.

Here we are able to obtain a universal negative as a conclusion, but not a universal affirmative. It is interesting to notice that the first figure alone enables us to prove a proposition of the form A.

It may also be pointed out that the combination IE, although not excluded by the general rules of the syllogism, cannot be used at all as premises, since it violates the canons of all four figures. There remain in all, then, nineteen valid moods of the syllogism, — four in the first figure, four in the second, six in the third, and five in the fourth figure.

§ 34. **The Mnemonic Lines.** — It is not necessary to commit to memory the valid moods in each figure. By applying the general rules of the syllogism to the figure in question, the student will be able to determine for himself in every case whether or not an argument is valid. The Latin Schoolmen in the thirteenth century, however, invented a system of curious mnemonic verses for the purpose of rendering it easy to remember the valid moods in each figure. Although it is not necessary for the student to burden his memory with these barbarous names, it is interesting to understand the use of the lines: —

*Barbara, Celarent, Darii, Ferio*que prioris;
Cesare, Camestres, Festino, Baroko, secundæ;
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bokardo, Ferison, habet; Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

The words printed in ordinary type are real Latin

words, indicating that the four moods represented by Barbara, Celarent, Darii, and Ferio are the valid moods of the first figure, that the next four are valid in the second figure, that the third figure has six valid moods represented by as many artificial names, and that the fourth figure adds five more. Each word represents a mood, the vowels A, E, I, and O indicating the quality and quantity of the propositions which go to compose them. Thus, Barbara signifies the mood of the first figure which is made up of three universal affirmative propositions A A A; Cesare, a mood of the second figure, composed of the three propositions E A E. These lines, then, sum up the results reached on pages 120–22 regarding the valid moods in each figure.

But certain consonants in these mnemonic words also indicate how arguments in the second, third, or fourth figures may be changed to the form of the first figure. The first figure was called by Aristotle the **perfect figure**, and the second and third the **imperfect figures**, since he did not regard an argument in these forms as so direct and convincing as one of the first-mentioned type. The fourth figure was not recognized by Aristotle, but is said to have been introduced into logic by Galen, the celebrated teacher of medicine, who lived in the latter half of the second century. The process of changing an argument from one of the so-called imperfect figures to that of the first figure is known as **Reduction**. And, as we have said, these curious but ingenious mnemonic words give rules for carrying out this process. For example, *s* indicates that the proposition represented by the preceding vowel is to be converted simply. Thus

an argument in the second figure of the mood Cesare is changed to Celarent in the first figure, by converting the major premise simply. Again, *p* denotes that the preceding vowel is to be converted by limitation, or *per accidens*; *m* is supposed to stand for *mutare*, and indicates that the premises are to be transposed; *k*, which is used in the moods Baroko and Bokardo, shows that an indirect method of proof or reduction is necessary to reduce the arguments to the first figure.

Further, the initial consonants of the moods of the imperfect figures correspond with those of the moods in the first figures, to which they can be reduced. Cesare and Camestres of the second figure, for example, and Camenes of the fourth are reducible to Celarent; and, similarly, Festino, Felapton, Fesapo, and Fresison may all be reduced to Ferio.

The student who understands the structure of the syllogism will be able to arrange an argument in one figure or another, as may be most convenient, without the aid of any mechanical rules. It may be interesting, however, to give a single example for the sake of illustrating the workings of this most ingenious device. Let us take the following argument in the second figure of the mood AEE, or Camestres: —

> All members of the class are prepared for the examination,
> No idle persons are prepared for the examination,
>
> Therefore no idle persons are members of the class.

Now the *m* in Camestres shows that the major and minor premises are to be transposed; the first *s* indicates that the minor premise is to be converted, and the second that the same process must be performed on the conclusion.

Converting the minor premise and transposing, we obtain: —

No persons prepared for the examination are idle,
All members of the class are prepared for the examination,
Converting the conclusion,
Therefore no members of the class are idle persons.

This result, as will at once be seen, is an argument in the first figure of the mood EAE, or Celarent.

References

Sir W. Hamilton, *Lectures on Logic.* Lectures XX., XXI.
A. Bain, *Logic*, Part First, Deduction, Bk. II. Ch. I.

NOTE. — It would be interesting to work out, in connection with the various forms of Inductive reasoning treated in Part II., the organic relation of the syllogistic Figures, and their natural applicability to various purposes of argument. This task, however, seemed to lie beyond the proper limits of this book. All of the investigations on this point start from Hegel's treatment in the second part of the *Wissenschaft der Logik* (*Werke*, Bd. 5, pp. 115 ff.). Those interested in this subject may consult W. T. Harris, *The Psychologic Foundations of Education*, Ch. IX.–XI., and the same author's *Logic of Hegel.* See also B. Bosanquet, *Logic*, Vol. II., pp. 44 ff., 88 ff., and *The Essentials of Logic*, Lecture X.

CHAPTER X

ABBREVIATED AND IRREGULAR FORMS OF ARGUMENT

§ 35. **Enthymemes.** — The term 'enthymeme' seems to have been used by Aristotle for an argument from signs or from likelihood, without complete proof. From this sense of logical incompleteness, the name has come to be applied in modern times to an argument in which some part is omitted. We have already noticed, in dealing with the syllogism (§ 10), that one premise is often omitted. Indeed, it is but seldom in ordinary reasoning that we arrange our arguments in the strict syllogistic form. We hurry on from one fact to another in our thinking without stopping to make all the steps definite and explicit. We feel it to be a waste of time, and a trial to the patience, to express what is clearly obvious, and so we press on to the conclusion which is, for the time being, the central point of interest.

But the more rapid and abbreviated the reasoning, the more necessary is it to keep a clear head, and to understand what conclusion is aimed at, and what premises are assumed in the argument. To bring to light the hidden assumption upon which an argument is based, is often the best means of refuting it.

Enthymemes are sometimes said to be of the first, second, or third order, according as the major premise, the minor premise, or the conclusion is wanting. As a matter of fact, an enthymeme of the third order is a rhetorical device used to call special attention to a conclusion which is perfectly obvious, although suppressed. Thus, for example, 'all boasters are cowards, and we have had proofs that A is a boaster.' Here the conclusion is at once obvious, and is even more prominent than if it were actually expressed.

It is usually easy to complete an enthymeme. If the conclusion and one premise are given, the three terms of the syllogism are already expressed. For the conclusion contains the major term and the minor term; and one of these again, in combination with the middle term, is found in the given premise. From these data, then, it will not be difficult to construct the suppressed premise. When the premises are given without the conclusion, there is no way of determining, except from the order, which is major and which is minor. It is therefore necessary to assume that they are already arranged in proper logical order, and that the subject of the conclusion, or minor term, is to be found in the second premise, and the predicate of the conclusion, or major term, in the first premise.

§ 36. **Prosyllogisms and Episyllogisms.** — In deductive reasoning it is often necessary to carry on the argument through several syllogisms, using the conclusion first reached as a premise in the following syllogism. For example, we may argue: —

All B is A
All C is B
———
All C is A.
But all D is C
———
All D is A.

It is clear that we have here two arguments in the first figure. The first is called the **Prosyllogism**, and the latter the **Episyllogism**. If the argument were carried on further, so as to include three or more syllogisms, the second would form the Prosyllogism with respect to the third, while the third would be the Episyllogism of the second. A concrete example of this kind of reasoning may now be given: —

1. All timid men are suspicious,
All superstitious men are timid,
———
2. Therefore all superstitious men are suspicious.
But some educated men are superstitious,
———
Therefore some educated men are suspicious.

It will be noticed that in these examples the argument advances from the premises of the Prosyllogism, to the conclusion of the Episyllogism. It proceeds, that is to say, in a forward direction, developing the consequences of the premises which form its starting-point. This mode of investigation is therefore called the *Progressive* or *Synthetic*, since it goes steadily forward building up its results as it advances. To state the same thing in different words, we may say that the *Progressive* or *Synthetic* method advances from the conditions to what is conditioned, from causes to effects.

But it is often necessary to proceed in the opposite way. We have often to go back and show the grounds upon which our premises rest, instead of going forward to show what consequences follow from them. And when we do this we proceed *Regressively* or *Analytically*. To take an example which will illustrate both ways of proceeding: —

No man is infallible, for no man is omniscient
Aristotle was a man,

Therefore Aristotle was not infallible.

In advancing from the premises to the conclusion in this argument our procedure is progressive or synthetic. Instead of reasoning out the consequences of the premises, however, we may go back and show the grounds upon which the major premise rests. It is evident that this premise is itself the conclusion of a syllogism which may be expressed as follows: —

All infallible beings are omniscient,
No man is omniscient,

Therefore no man is infallible.

The regressive method goes backward from conclusions to premises, or from the conditioned to its necessary conditions. In scientific investigation it reasons from effects to causes, while the synthetic method advances from causes to effects.

§ 37. **Sorites, or Chains of Reasoning.** — A Sorites is an abbreviated form of syllogistic reasoning in which a subject and predicate are united by means of several intermediate terms. Such a train of reasoning represents several acts of comparison, and therefore several syllogistic steps. But instead of stopping to draw the conclusion at each stage, the sorites continues the processes of comparison, and only sums up its results at the close. We may define the sorites, therefore, as a series of prosyllogisms and episyllogisms in which all of the conclusions, except the last, are suppressed. It is usually stated in the following form: —

All A is B
All B is C
All C is D
All D is E

∴ All A is E.

It is evident that this train of reasoning fully expressed is equivalent to the following three syllogisms: —

First Syllogism	Second Syllogism	Third Syllogism
All B is C	All C is D	All D is E
All A is B	All A is C (1)	All A is D (2)
∴ All A is C (1).	∴ All A is D (2).	∴ All A is E (3).

There are two rules to be observed in using this form of the sorites: (1) The first premise may be particular, all the others must be universal; (2) the last premise may be negative, all the others must be affirmative. It is evident from an examination of the syllogisms given above that if any premise except the first were particular, the fallacy of undistributed middle would be committed. For, in that case, the middle term in one of the syllogisms would be the subject of a particular proposition, and the predicate of an affirmative proposition. And if any premise but the last were negative, the major term in the syllogism following that in which this occurred would be ~~disturbed~~ in the conclusion without having been distributed in the major premise. We may now give some concrete examples of this kind of reasoning: —

Misfortunes sometimes are circumstances tending to improve the character,
Circumstances tending to improve the character are promoters of happiness,
What promotes happiness is good,
Therefore misfortunes are sometimes good.

In some cases the different terms of an argument of this kind are expressed in the form of hypothetical

propositions. Thus, for example, we might argue: If a man is avaricious, he desires more than he possesses; if he desires more than he possesses, he is discontented; if he is discontented, he is unhappy; therefore if a man is avaricious, he is unhappy. This argument is hypothetical in form only, and may be easily reduced to categorical type as follows: —

An avaricious man is one who desires more than he possesses,
A man who desires more than he possesses is discontented,
A discontented man is unhappy,

Therefore an avaricious man is unhappy.

It will be noticed that the subject of the first premise in this form of argument is taken as the subject of the conclusion, and that the predicate of the conclusion is the predicate of the last premise. This is usually called the Aristotelian sorites. But there is another form which unites in the conclusion the subject of the last premise, and the predicate of the first, and which is known as the Goclenian sorites.[1] This may be thus represented: —

All A is B
All C is A
All D is C
All E is D

∴ All E is B.

Since B is the predicate of the conclusion, the premise in which it appears is always to be regarded as the major. As a result of this, it is to be noticed that the

[1] Rudolf Goclenius (1547–1628), Professor at Marburg, first explained this form in his *Isagoge in Organum Aristotlis*, 1598.

suppressed conclusions in this argument form the major premise of the following syllogism, instead of the minor premise as in the Aristotelian sorites. We may, therefore, expand the reasoning into the three following syllogisms: —

First Syllogism	Second Syllogism	Third Syllogism
All A is B	All C is B	All D is B
All C is A	All D is C	All E is D
∴ All C is B.	∴ All D is B.	∴ All E is B.

A little consideration of the form of these syllogisms will lead the student to see that the rules given for the Aristotelian sorites must be here reversed. In both forms of the sorites there cannot be more than one negative premise, nor more than one particular premise. In the Aristotelian form, no premise except the last can be negative, and no premise except the first particular. In the Goclenian sorites, on the other hand, the single premise which can be negative is the first, and it is the last alone which may be particular.

§ 38. **Irregular Arguments.** — There are a large number of arguments employed in everyday life which are valid and convincing, and yet which cannot be reduced to the syllogistic form. The difficulty with these arguments is that they appear to have four terms, at least in the form in which they are most naturally stated. We may discuss such irregular forms of reasoning under two headings: (1) Arguments which deal with the relations of things in time and space, or with their quantitative determinations; (2) arguments which are

largely verbal in character, and may be said to depend upon the principle of substitution.

(1) As an example of the first class of argument we may take the following: —

A is greater than B,
B is greater than C,
Therefore A is still greater than C.

It is obvious that, although we have here four terms, the conclusion is valid, and the form of argument perfectly convincing. The truth seems to be that in reasoning about quantities we do not proceed upon the syllogistic principle of the inclusion and exclusion of terms. But knowing the continuous nature of quantity, we take as our principle that, 'what is greater than that which is greater than another is *a fortiori* greater than that other.' It would not, however, make the matter any clearer to write this as our major premise, and bring the real argument under it in this way: —

What is greater than that which is greater than another is still greater than that other,
A is that which is greater than that which is greater than C,
Therefore A is still greater than C.

What we have here given as the major premise is simply a statement of the nature of quantity, not a premise from which the conclusion is derived. We find the same irregularity in arguments referring to the relations of things in space and time: —

A is situated to the east of B,
B is situated to the east of C,
Therefore A is to the east of C.

In spite of the formal deficiency of four terms the argument is valid. It will be observed, too, that it is in virtue of the comparison of the position of A and of C with that of B, that these relative positions have been determined. The principle upon which we proceed may be said to be that, 'what is to the east of B is to the east of that which B is to the east of.' Or perhaps it would be truer to fact to say that we proceed in such cases upon what we know regarding the nature of space, and the relations of objects in space.

(2) The second class of irregular arguments are largely verbal in character, and may be dealt with very briefly. As an example we may consider: —

Men are willing to risk their lives for gold,
Gold cannot buy happiness,

Therefore men are willing to risk their lives for what cannot buy happiness.

It is doubtful, I think, whether these propositions represent any real inference. The whole process may be regarded as a verbal substitution in the major premise of 'what cannot buy happiness' for the word 'gold.' By a slight change in the form of the proposition, however, the argument may be expressed as a regular syllogism of the third figure: — p. 114

Gold is something for which men are willing to risk their lives,
Gold cannot buy happiness,

Therefore something which cannot buy happiness is something for which men are willing to risk their lives.

Another example which also appears to be irregular at first sight is added: —

The men of the Middle Ages were ready to undertake any expedition where glory could be won,

The crusades were expeditions in which glory could be won,

The crusades, therefore, were readily undertaken by the men of the Middle Ages.

This argument seems to be irregular in form only, and by a slight change in form may be expressed in the first figure: — p. 114

All expeditions in which glory could be won were readily undertaken by the men of the Middle Ages,

The crusades were expeditions in which glory could be won,

Therefore the crusades were readily undertaken by the men of the Middle Ages.

References, especially for § 38

W. S. Jevons, *Elementary Lessons in Logic*, p. 152.

" " " *The Principles of Science*, Introduction.

F. H. Bradley, *The Principles of Logic*, pp. 348–360.

CHAPTER XI

HYPOTHETICAL AND DISJUNCTIVE ARGUMENTS

§ 39. **The Hypothetical Syllogism.** — We have hitherto been dealing with syllogisms composed entirely of categorical propositions, and have not referred to the use which is made of conditional propositions in reasoning. A conditional proposition is sometimes defined as the union of two categorical propositions by means of a conjunction. It is the expression of an act of judgment which does not directly or unambiguously assert something of reality. We have already pointed out (§ 20) that there are two classes of conditional propositions: the *hypothetical* and the *disjunctive*, and corresponding to these we have the **hypothetical** and the **disjunctive syllogism**. The hypothetical syllogism has a hypothetical proposition as a major premise, and a categorical proposition as a minor premise. The disjunctive syllogism in the same way is composed of a disjunctive proposition as major, and a categorical proposition as minor, premise. In addition to these, we shall have to treat of another form of argument called the 'dilemma,' which is made up of hypothetical and disjunctive propositions.

A hypothetical proposition asserts something not directly, but subject to some limitation or condition. It is usually introduced by some word or conjunctive

phrase, like 'if,' 'supposing,' or 'granted that'; as, *e.g.*, 'if he were to be trusted, we might give him the message'; 'suppose that A is B, then C is D.' The part of a hypothetical proposition which expresses the supposition or condition is known as the **Antecedent**; the clause stating the result is called the **Consequent.** Thus, in the proposition, 'he would write if he were well,' the consequent, 'he would write,' is stated first, and the antecedent, 'if he were well,' follows.

The hypothetical syllogism, as has been already remarked, has a hypothetical proposition as its major, and a categorical proposition as its minor, premise: —

> If justice is to prevail, his innocence will be proved,
> And justice will prevail,
>
> Therefore his innocence will be proved.

It will be noticed that in this argument the minor premise *affirms the antecedent,* and that, as a result, the conclusion affirms the consequent. This form is known as the *constructive hypothetical syllogism*, or the *modus ponens.*

In the following example it will be observed that the *consequent is denied*, and the conclusion obtained is therefore negative.

> If he were well, he would write,
> He has not written,
>
> Therefore he is not well.

This is called the *destructive hypothetical syllogism*, or *modus tollens.*

The rule of the hypothetical syllogism may therefore be stated as follows: *Either affirm the antecedent or*

deny the consequent. If we affirm the antecedent, *i.e.*, declare that the condition exists, the consequent necessarily follows. And, on the other hand, if the consequent is declared to be non-existent, we are justified in denying that the condition is operative.

The violation of these rules gives rise to the fallacies of *denying the antecedent*, and of *affirming the consequent.* Thus, for example, we might argue: —

> If he were well, he would write,
> But he is not well,
> ___
> Therefore he will not write.

Here the antecedent is denied, and the argument plainly false. For we cannot infer that his being well is the only condition under which he would write. We do not know, in other words, that the antecedent stated here is the only, or essential condition of the consequent. We know that if there is fire, there must be heat; but we cannot infer that there is no heat when no fire is present. Of course, if we can be certain that our antecedent expresses the essential condition, or real *sine qua non* of the consequent, we can go from the denial of the former to that of the latter. For example: —

> If a triangle is equilateral, it is also equiangular,
> This triangle is not equilateral,
> ___
> Therefore it is not equiangular.

Usually, however, when the hypothetical form of expression is employed, we cannot be certain that the antecedent expresses the sole, or essential condition, of the consequent. At the ordinary stages of knowledge

we have to content ourselves with reasoning from antecedent conditions, without being able to show that no other condition is possible.

To illustrate the fallacy of affirming the consequent, we may take the following example: —

If perfect justice prevailed, the rich would not be permitted to rob the poor,
But the rich are not permitted to rob the poor,

Therefore perfect justice prevails.

Here it will be noticed that the consequent states only *one* result of the prevalence of 'perfect justice.' Because the consequent is declared to exist, it by no means follows that it exists *as a consequence* of the operation of this condition. It is also worth noting in this example that the consequent of the major premise is negative. The minor premise *which affirms the consequent* also takes a negative form. To deny the consequent we should have to say, 'the rich are permitted to rob the poor.' Or, to put the matter generally, it is necessary to remember that the affirmation of a negative proposition is expressed by a negative proposition, and that the denial of a negative — the negation of a negation — is, of course, positive in form.

§ 40. **Relation of Categorical and Hypothetical Arguments.** — It is evident that the form of the hypothetical syllogism is very different from that of the categorical. But, although this is the case, it must not be supposed that with the former we have passed to a new and wholly distinct type of reasoning. In hypothetical

reasoning, as in categorical, it is the presence of a universal principle which enables us to bring two facts into relation which formerly stood apart. Indeed, in many cases, it is a matter of indifference in which form the argument is stated. Thus, we may argue in hypothetical form: —

> If a man is industrious, he will be successful,
> A is an industrious man,
>
> Therefore A will be successful.

The same argument may, however, be expressed equally well in categorical form: —

> All industrious men will be successful,
> A is an industrious man,
>
> Therefore A will be successful.

It is clear that, in spite of the different forms in which the argument is expressed, the reasoning is essentially the same in both cases. The middle term, or general principle which makes it possible to unite the subject and predicate of the conclusion, in the hypothetical as well as in the categorical syllogism, is 'industrious.' A will be successful, we argue, because he is industrious, and it is a rule that industrious men are successful.

Moreover, if an argument is fallacious in one form, it will also be fallacious when expressed in the other. The defects of an argument cannot be cured simply by a change in its form. When a hypothetical argument, in which the antecedent is denied, is expressed categorically, we have the fallacy of the illicit major term. Thus, to state the example of denying the antecedent given on page 138, we get: —

The case of his being well is a case of his writing,
The present is not a case of his being well,

Therefore the present is not a case of his writing.

Similarly, when an argument in which the consequent is affirmed is changed to the categorical form, the defect in the reasoning appears as the fallacy of undistributed middle: —

If this tree were an oak, it would have rough bark and acorns,
This tree has rough bark and acorns,

Therefore it is an oak.

When this argument is expressed in categorical form, it is at once clear that the middle term is not distributed in either the major or minor premise: —

All oak trees are trees having rough bark and acorns,
This tree is a tree having rough bark and acorns,

Therefore this tree is an oak.

The change from the categorical to the hypothetical form of argument, then, does not imply any essential change in the nature of the reasoning process itself. Nevertheless, it is important to note that hypothetical propositions and hypothetical arguments emphasize one aspect of thinking, which is entirely neglected by the theory of the categorical syllogism. When dealing with the extension of terms (§ 16), we pointed out that every term, as actually used in a proposition, has both an extensive and an intensive function. That is, the terms of a proposition are employed both to name certain objects or groups of objects, and to connote or imply certain attributes or qualities. In the proposition, 'these are oak trees,' the *main* purpose is to identify the trees

given in perception with the class of oak trees. When, on the other hand, we say, 'ignorant people are superstitious,' the proposition does not refer directly to any particular individuals, but states the necessary connection between ignorance and superstition. Although the existence of ignorant persons who are also superstitious is *presupposed* in the proposition, its most prominent function is to assert a connection of attributes which is wholly impersonal. We may perhaps say that, in spite of the categorical form, the proposition is essentially hypothetical in character. Its meaning might very well be expressed by the statement, 'if a man is ignorant, he is also superstitious.' What is here emphasized is not the fact that ignorant persons exist, and are included in the class of superstitious persons, but rather the general law of the necessary connection of ignorance and superstition. The existence of individuals to whom the law applies is, of course, presupposed by the proposition. It is not, however, its main purpose to directly affirm their existence.

We have reached, then, the following position: Every judgment has two sides, or operates in two ways. On the one hand, it asserts the existence of individual things, and sets forth their qualities and relations to other things. But, at the same time, every judgment seeks to go beyond the particular case, and to read off a general law of the connection of attributes or qualities which shall be true universally. In singular and particular propositions, the categorical element — the direct assertion of the existence of particular objects — is most prominent, although even here the hint or suggestion

of a general law is not altogether absent. When we reach the universal proposition, however, the reference to real things is much less direct, and the meaning seems capable of expression in hypothetical form.

Now in the chapters on the categorical syllogism this latter aspect of judgments has been left out of account. Propositions were there interpreted as referring directly to objects, or classes of objects (cf. § 23). The proposition, S is P, for example, was taken to affirm that some definite object, or class of objects, S, falls within the class P. And the fact that it is possible to apply this theory shows that it represents one side of the truth. But the student must sometimes have felt that, in this procedure, the most important signification of the proposition is lost sight of. It seems absurd to say, for example, that in the proposition, 'all material bodies gravitate,' the class of 'material bodies' is included in the wider class of 'things that gravitate.' The main purpose of the judgment is evidently to affirm the necessary connection of the attributes of materiality and gravitation. The judgment does not refer directly to things, or classes of things at all, but asserts without immediate reference to any particular object, *if* material, *then* gravitating. The propositions of geometry are still more obviously hypothetical in character. 'The three angles of a triangle are equal to two right angles,' for example, cannot, without violence, be made to mean that the subject is included in the class of things which are equal to two right angles. The main purpose of the proposition is obviously to assert the necessary connection of

the 'triangularity' and the equality of angles with two right angles, and not to make any direct assertion regarding any actually existing object or group of objects.

We reach, then, the following conclusion: Our thought is at once both categorical and hypothetical. As categorical, it refers directly to objects and their relations. The terms of the proposition are then taken in extension to represent objects or groups of objects, and the copula to assert the inclusion of the subject in the predicate, or, in cases of negative propositions, to deny this relation. As hypothetical, the reference to things is much more indirect. The terms of the proposition are no longer regarded as representing objects or classes, but are interpreted from the point of view of *intension*. The judgment affirms or denies the connection of the qualities or attributes *connoted* by the terms, and not that of the objects which they denote. Sometimes the one aspect of thought, sometimes the other, is most prominent.

In sense-perception and in simple historical narration, assertions are made directly and categorically regarding things and events. The main interest is in particular objects, persons, or events, and our judgments refer directly and unambiguously to them. But, as we have already seen, our thought from its very beginning attempts to get beyond the existence of particular things and events, and to discover what qualities of objects are necessarily connected. We pass from perception and observation to explanation, from the narration of events, to the discovery of the law of their connection. And,

as a result of this advance, our judgments deal no longer exclusively with particular objects and events, and the fact of their relation, but with the general laws of the connection between attributes and qualities. There is, of course, no fixed point at which we pass from the categorical to the hypothetical aspect of thinking. But, in general, as we pass from judgments of sense-perception and memory, to a statement of theories and laws, the hypothetical element comes more and more clearly into the foreground. We have seen that it is almost impossible to interpret propositions regarding geometrical relations as referring directly to classes of objects. In the same way, it is evident that propositions which state general laws are more truly hypothetical than categorical. When we assert that 'all men are mortal,' the proposition does not intend to state a fact in regard to each and every man, or to refer directly to individuals at all, but to express the essential and necessary relation between humanity and mortality. A proposition which is essentially hypothetical in character, may then be expressed in categorical form. It must be remembered that it is not the form, but the purpose or function of a proposition, which determines its character. The hypothetical form, however, does justice to an aspect of thought which is especially prominent in the universal laws and formulas of scientific knowledge, and which is not adequately represented by the theory of subsumption, or the inclusion of the subject in the predicate.

§ 41. **Disjunctive Arguments.** — A disjunctive proposition, as we have already seen, is of the form, 'A is

either B, or C, or D'; or, when expressed negatively, 'A is neither B, nor C, nor D.' It is sometimes said to be the union of a categorical and a hypothetical proposition. On the one hand, it asserts categorically regarding A, and without reference to any external condition. But the disjunctive proposition is not simple like the categorical proposition: it states its results as a series of related conditions and consequences. If A is not B, it tells us, it must be either C or D; and if it is C, it follows that it cannot be B or D.

A disjunctive proposition may at first sight appear to be a mere statement of ignorance, and, as such, to be less useful than the simple categorical judgment of perception. And it is true that the disjunctive form may be employed to express lack of knowledge. 'I do not know whether this tree is an oak or an ash'; 'he will come on Monday or some other day.' A true disjunctive proposition, however, is not a mere statement of ignorance regarding the presence or absence of some fact of perception. It is an attempt, on the part of intelligence, to determine the whole series of circumstances or conditions within which any fact of perception may fall, and to state the conditions in such a way that their relations are at once evident. And to do this implies positive knowledge. In the first place, the enumeration of possibilities must be exhaustive, no cases must be overlooked, and no circumstances left out of account. Secondly, the members of the proposition must be taken so as to be really disjunctive. That is, they must be exclusive of one another. We cannot combine disjunctively any terms we please

with each other. But it is only when we understand the systematic connections of things in the field in question, that we are able to express them in the form *either* B *or* C, and thus assert that the presence of one excludes the other.

A disjunctive proposition, then, presupposes systematic knowledge, and is consequently the expression of a comparatively late stage in the evolution of thought. It is true that disjunction may involve doubt or ignorance regarding any particular individual. We may not be able to say whether A is B or C or D. But, before we can formulate the disjunctive proposition, we must be already acquainted with the whole set of possible conditions, and also with the relation in which those conditions stand to each other. Our knowledge, when formulated in the disjunctive major premise of an argument, is so exhaustive and systematic, that the application to a particular case effected by the minor premise appears almost as a tautology. This will be evident in the disjunctive arguments given below.

There are two forms of the disjunctive syllogism. The first is sometimes called the *modus tollendo ponens*, or the mood which affirms by denying. The minor premise, that is, is negative, and the conclusion affirmative. The form is, —

A is either B or C,
A is not C,
―――――――――
Therefore A is B.

The negative disjunctive argument has an affirmative minor premise. It is known as the *modus ponendo*

tollens, or the form which, by affirming one member of the disjunctive series, denies the others, —

A is B or C or D,
But A is B,
Therefore A is neither C nor D.

It is, of course, a very simple matter to draw the conclusion from the premises in these cases. As we have already indicated, the real intellectual work consists in obtaining the premises, especially in discovering the relations enumerated in the major premise. It is in formulating the major premise, too, that errors are most likely to arise. As already pointed out, it is essential that the disjunctive members shall be exhaustively enumerated, and also that they shall exclude each other. But it is not always easy to discover all the possibilities of a case, or to formulate them in such a way that they are really exclusive. If we say, 'he is either a knave or a fool,' we omit the possibility of his being both the one and the other to some extent. A great many statements which are expressed in the form of disjunctive propositions are not true logical disjunctives. Thus we might say, 'every student works either from love of learning, or from love of praise, or for the sake of some material reward.' But the disjunction does not answer the logical requirements, for it is possible that two or more of these motives may influence his conduct at the same time. The disjunctive members are neither exclusive nor completely enumerated.

§ 42. **The Dilemma.** — A dilemma is an argument composed of hypothetical and disjunctive propositions.

As the word is used in ordinary life, we are said to be in a dilemma whenever there are but two courses of action open to us; and when both of these have unpleasant consequences. In the same way, the logical dilemma shuts us in to a choice between alternatives, either of which leads to a conclusion we would gladly avoid.

The first form, which is sometimes called the **Simple Constructive Dilemma**, yields a simple or categorical conclusion, —

> If A is B, C is D; and if E is F, C is D,
> But either A is B, or E is F,
>
> Therefore C is D.

It will be noticed that the minor premise affirms disjunctively the antecedents of the two hypothetical propositions which form the major premise, and that the conclusion follows whichever alternative holds. We may take as a concrete example of this type of argument: —

> If a man acts in accordance with his own judgment, he will be criticised; and if he is guided by the opinions and rules of others, he will be criticised.
>
> But he must either act in accordance with his own judgment, or be guided by the opinions of others.
>
> Therefore, in any case, he will be criticised.

The hypothetical propositions which make up the major premise of a dilemma do not usually have the same consequent, as is the case in the examples just given. When the consequents involved are different, the dilemma is said to be complex, and the conclusion has the form of a disjunctive proposition. In the **Complex**

Constructive Dilemma, the minor premise affirms disjunctively the antecedents of the major, and the conclusion is consequently affirmative. We may take, as an example, the argument by which the Caliph Omar is said to have justified the burning of the Alexandrian library: —

If these books contain the same doctrines as the Koran, they are unnecessary; and if they are at variance with the Koran, they are wicked and pernicious.

But they must either contain the same doctrines as the Koran or be at variance with it.

Therefore these books are either unnecessary or wicked and pernicious.

A third form, the **Complex Destructive Dilemma**, obtains a negative disjunctive proposition as a conclusion, by denying the consequents of the hypothetical propositions which form the major premise of the argument. We may take the following example: —

If a man is prudent, he will avoid needless dangers; if he is bold and courageous, he will face dangers bravely.

But this man neither avoids needless dangers nor does he face dangers bravely.

Therefore he is neither prudent nor bold and courageous.

By taking more than two hypothetical propositions as major premise, we may obtain a Trilemma, a Tetralemma, or a Polylemma. These forms, however, are used much less frequently than the Dilemma.

The dilemma is essentially a polemical or controversial form of argument. Its object, as we have seen, is to force an unwelcome conclusion upon an adversary, by showing that his argument, or his conduct, admits of

one or other of two unpleasant interpretations. We sometimes speak of the horns of the dilemma, and of our adversary as 'gored,' whichever horn he may choose. Dilemmas, however, like all controversial arguments, are more often fallacious than valid. The minor premise of a dilemmatic argument, as we have already seen, is a disjunctive proposition with two members. But it is ~~very rarely~~ that two alternatives exhaust all the possible cases. The cases enumerated, too, may not exclude each other, or be real alternatives at all. The dilemma is thus subject to all the dangers which we have already noticed in the case of the disjunctive argument. In addition, it is necessary to see that the canon of the hypothetical syllogism, 'affirm the antecedent or deny the consequent,' is observed. If this rule is not observed, the logical form of the argument will not be correct.

References, especially for § 40

J. S. Mill, *Logic*, Bk. I. Ch. V.

C. Sigwart, *Logic*, Pt. I. Ch. VII.

W. Minto, *Logic Inductive and Deductive*, pp. 129–138, and pp. 214–225.

F. H. Bradley, *The Principles of Logic*, Bk. I. Ch. 2.

B. Bosanquet, *The Essentials of Logic*, Lecture VI.

CHAPTER XII

FALLACIES OF DEDUCTIVE REASONING

§ 43. **Classification of Fallacies.** — We shall hereafter treat of the fallacies or errors to which inductive reasoning is most subject (Ch. xix.). At present, however, it is necessary to consider the fallacies which are likely to attend the employment of the syllogistic form of reasoning. In considering the subject, we shall find that many fallacies belong equally to both kinds of reasoning. This is especially true of errors which arise from the careless use of words.

The first systematic account of fallacies is given in Aristotle's treatise, *On Sophistical Difficulties* (περὶ σοφιστικῶν ἐλέγχων). In this work, Aristotle divides fallacies into two classes: those which are due to language (παρὰ τὴν λέξιν, or, as they are usually called, fallacies *in dictione*), and those which are not connected with language (ἔξω τῆς λέξεως, *extra dictionem*). Under the first head, he enumerates six kinds of fallacies, and under the second, seven. Aristotle's principle of classification is, however, not entirely satisfactory. We must try to find some positive principle or principles of classification which will render us more assistance in understanding the relations between the various fallacies than is afforded by Aristotle's division into those which belong to language, and those which do not.

In the strict sense of the word, a fallacy is to be defined as an error in reasoning. In the syllogism, however, propositions or premises form the data or starting-point. If, now, these propositions are not properly understood, the conclusions to which they lead are likely to be false. We may then first divide fallacies into **Errors of Interpretation,** and **Fallacies in Reasoning.** Errors in interpreting propositions might, perhaps, be more properly treated in a work on rhetoric than in a chapter on logical fallacies. But it has been the custom ever since the time of Aristotle to include in the enumeration of logical fallacies a number of errors which are likely to arise in interpreting propositions. Moreover, as we saw in Chapter VII., there are certain processes of interpretation, like Obversion and Conversion, which are sometimes called immediate inference, and which require a knowledge of the logical structure of propositions.

The Fallacies which arise in the process of reasoning, we may again divide into **Formal Fallacies,** or violations of the syllogistic rules, and **Material Fallacies.** The latter class may be further divided into **Fallacies of Equivocation** (including Ambiguous Middle, Composition, Division, and Accident) and **Fallacies of Presumption** (including Petitio Principii, Irrelevant Conclusion, Non Sequitur, and Complex Questions). The following table will summarize this classification: —

FALLACIES

- *Errors in Interpretation*
  - (1) Illogical Obversion or Conversion
  - (2) Amphiboly
  - (3) Accent
- *Mistakes in Reasoning*
  - *Formal*
    - In Categorical Arguments
      - (1) Four Terms
      - (2) Undistributed Middle
      - (3) Illicit Major
      - (4) Illicit Minor
      - (5) Negative Premises
    - In Hypothetical Arguments
      - (6) Denying the Antecedent
      - (7) Affirming the Consequent
    - In Disjunctive Arguments
      - (8) Imperfect Disjunction
  - *Material*
    - *Equivocation*
      - (1) Ambiguous Middle
      - (2) Composition
      - (3) Division
      - (4) Accident
    - *Presumption*
      - (1) Petitio Principii
      - (2) Complex Question
      - (3) Irrelevant Conclusion
      - (4) Non Sequitur

§ 44. **Errors in Interpretation.** — This class of fallacies results from imperfect understanding of the meaning of propositions. They are not, then, strictly speaking, errors of reasoning at all. If, however, the propositions employed as premises in an argument are not correctly understood, the conclusions founded upon them are likely to be erroneous. And even if the proposition, which is wrongly interpreted, is not made the basis of further reasoning, it is in itself the result of an intellectual error against which it is possible to guard. We do not, of course, profess to point out all the possible sources of error in interpreting propositions. The only

rule applicable to all cases which can be given is this: Accept no proposition until you understand its exact meaning, and know precisely what it implies. Deliberation and attention, both with regard to our own statements and those of others, are the only means of escaping errors of this kind.

(1) *Illogical Obversion or Conversion.* — In a previous chapter (Ch. vii.), we have treated of Obversion and Conversion, and shown the rules to be followed in stating the obverse or the converse of a proposition. In Obversion, we interpret or show what is involved in a proposition, by stating its implications in a proposition of the opposite quality. And unless we have clearly grasped the meaning of the original proposition, mistakes are likely to arise in changing from the affirmative to the negative form of statement, or from the negative to the affirmative. Thus, we should fall into an error of this kind if we should take the proposition, 'honesty is always good policy,' to be the equivalent of, or to imply, the statement, 'dishonesty is always bad policy.' Nor can we obtain by obversion the proposition, 'all citizens are allowed to vote,' from, 'no aliens are allowed to vote.'

In Conversion, we take some proposition, A is B, and ask what assertion it implies regarding the predicate. Does 'all brave men are generous' imply also that 'all generous men are brave'? This is, perhaps, the most frequent source of error in the conversion of propositions. I do not mean that in working logical examples we are likely to convert proposition A simply, instead of by limitation. But in the heat of debate, or when using

propositions without proper attention, there is a natural tendency to assume that a proposition which makes a universal statement regarding the subject, does the same with regard to the predicate. And, although such errors are very obvious when pointed out, — as, indeed, is the case with nearly all logical fallacies, — they may very easily impose upon us when our minds are not fully awake, that is, when attention is not active and consciously on guard. Of the other methods of conversion perhaps contraposition is most likely to be a source of error. We have already (§ 27) given the rules for obtaining the contrapositive of any proposition. Some practice in working examples will assist students in perceiving what is the logical contrapositive to any proposition, and in detecting fallacies.

(2) *Amphiboly*, or amphibology (ἀμφιβολία), consists in misconception arising from the ambiguous grammatical construction of a proposition. A sentence may have two opposite meanings, but one may be more natural and prominent than the other. A deception may be practised by leading a person to accept the meaning more strongly suggested, while the significance intended is the very opposite, as, *e.g.*, 'I hope that you the enemy will slay.' In Shakespeare's *Henry VI.*, we have an instance of amphiboly in the prophecy of the spirit, that "the Duke yet lives that Henry shall depose."

(3) The *Fallacy of Accent* is a misconception due to the accent or emphasis being placed upon the wrong words in a sentence. It may, therefore, be regarded as a rhetorical, rather than as a logical fallacy. Jevons's

examples of this fallacy may be quoted in part. "A ludicrous instance is liable to occur in reading Chapter XIII. of the First Book of Kings, verse 27, where it is said of the prophet, 'And he spake to his sons, saying, Saddle me the ass. And they saddled *him*.' The italics indicate that the word *him* was supplied by the translators of the authorized version, but it may suggest a very different meaning. The commandment, 'Thou shalt not bear false witness against thy neighbour,' may be made by a slight emphasis of the voice on the last word to imply that we are at liberty to bear false witness against other persons. Mr. De Morgan who remarks this also points out that the erroneous quoting of an author, by unfairly separating a word from its context, or italicizing words which were not intended to be italicized, gives rise to cases of this fallacy."[1] Jevons is also authority for the statement that Jeremy Bentham was so much afraid of being led astray by this fallacy that he employed a person to read to him whose voice and manner of reading were particularly monotonous.

§ 45. **Formal Fallacies.** — We shall follow our table, and deal with mistakes of Reasoning under the two headings of Formal Fallacies, and Material Fallacies. Formal fallacies arise from violations of the rules of the syllogism. The breaches of these rules have been already pointed out, and illustrated in our discussion of the various forms of syllogistic argument. The analysis of arguments, with a view to the detection of such fallacies, where any exist, is a very important exercise,

[1] Jevons, *Lessons in Logic*, p. 174.

and affords valuable mental discipline. It seems only necessary here to add a remark regarding the first fallacy on our list, that of Four Terms, or *Quaternio Terminorum*, as it is usually called by logicians.

The first canon of the categorical syllogism states that 'a syllogism must contain three and only three terms.' This rule would of course be violated by such an argument as, —

> Frenchmen are Europeans,
> Englishmen are Anglo-Saxons,
> Therefore Englishmen are Europeans.

It is so obvious that this example does not contain a real inference that no one would be likely to be misled by the pretence of argument which it contains. In some cases, however, a term may be used in two senses, although the words by which it is expressed are the same. The following example may be given: —

> Every good law should be obeyed,
> The law of gravitation is a good law,
> Therefore the law of gravitation should be obeyed.

Here we have really four terms. The word 'law,' in the first proposition, means a command given or enactment made by some persons in authority. A 'good law' in this sense then means a just law, or one which has beneficial results. But in the second proposition it signifies a statement of the uniform way in which phenomena behave under certain conditions. A 'good law' from this point of view would imply a correct statement of these uniformities. It is interesting to note that this example may also be regarded as an

instance of Equivocation, and classified as a case of an **ambiguous middle term.** It is often possible to classify a fallacy under more than a single head.

There are, however, cases where an argument may seem at first sight to have four terms, but where the defect is only verbal. The matter must, of course, be determined by reference to the meaning of terms and not merely to the verbal form of expression. It is ideas or concepts, and not a form of words, which are really operative in reasoning.

§ 46. **Material Fallacies.** — What are called material fallacies do not result from the violation of any specific logical rules. They are usually said to exist, not in the *form*, but in the *matter* of the argument. Consequently, it is sometimes argued, the detection and description of them do not properly belong to logic at all. We have found, however, that all these fallacies have their source in Equivocation and Presumption. They thus violate two of the fundamental principles of logical argument. For all logical reasoning presupposes that the terms employed shall be clearly defined, and used throughout the argument with a fixed and definite signification. And, secondly, logic requires that the conclusion shall not be assumed, but derived strictly from the premises. The violation of these principles is, therefore, a proper matter of concern to the logician. We shall treat first of the fallacies of Equivocation.

(*A*) The fallacies of Equivocation have been enumerated as Ambiguous Middle Term, Composition, Division, and Accident. These all result from a lack of clearness

and definiteness in the terms employed. We shall deal with them briefly in order.

(1) The phrase, **Ambiguous Middle Term**, describes the first fallacy of this group. It is obvious that the middle term cannot form a proper standard of comparison if its meaning is uncertain or shifting. A standard of measure must be fixed and definite. One illustration of this fallacy will be sufficient: —

Partisans are not to be trusted,
Democrats are partisans,
Therefore Democrats are not to be trusted.

The middle term, 'partisan,' is evidently used in two senses in this argument. In the first premise it signifies persons who are deeply or personally interested in some measure; and in the latter it simply denotes the members of a political party. When an argument is long, and is not arranged in syllogistic form, this fallacy is much more difficult of detection than in the simple example which has been given. It is of the utmost importance, then, to insist on realizing clearly in consciousness the ideas for which each term stands, and not to content ourselves with following the words.

(2) The fallacy of **Composition** arises when we affirm something to be true of a whole, which holds true only of one or more of its parts when taken separately or *distributively*. Sometimes the error is due to confusion between the distributive and collective signification of 'all,' as in the following example: —

All the angles of a triangle are less than two right angles,
A, B, and C are all the angles of this triangle,
Therefore A, B, and C are less than two right angles.

It is, of course, obvious that 'all the angles of a triangle' in the major premise signifies each and every angle when taken by itself, and that the same words in the minor premise signify all the angles collectively. What is true of all the parts taken separately, is not necessarily true of the whole. We cannot say that because no one member of a jury is very wise or very fair-minded, that the jury as a whole are not likely to bring in a just verdict. The members may mutually correct and supplement each other, so that the finding of the jury as a whole will be much fairer and wiser than the judgment of any single individual composing it. Another instance of this fallacy which is often quoted is that by which protective duties are sometimes supported:—

> The manufacturers of woollens are benefited by the duty on woollen goods; the manufacturers of cotton by the duty on cotton; the farmer by the duties on wool and grain; and so on for all the other producing classes; therefore, if all the products of the country were protected by an import duty, all the producing classes would be benefited thereby.

But, because each class would be benefited by an import tax upon some particular product, it does not necessarily follow that the community as a whole would be benefited if all products were thus protected. For, obviously, the advantages which any class would obtain might be more than offset by the increased price of the things which they would have to buy. On the other hand, it would be necessary to take into consideration the fact that an increase in the prosperity of one class indirectly brings profit to all the other members of the same society.

We cannot regard a whole as simply a sum of parts, but must consider also the way in which the parts act and react upon each other.

(3) The fallacy of **Division** is the converse of Composition. It consists in assuming that what is true of the whole is also true of the parts taken separately. Some term, which is used in the major premise collectively, is employed in a distributive sense in the minor premise and conclusion. The following example will illustrate this: —

> All the angles of a triangle are equal to two right angles,
> A is an angle of a triangle,
>
> Therefore A is equal to two right angles.

To argue that, because some measure benefits the country as a whole, it must therefore benefit every section of the country, would be another instance of this fallacy. Again, we may often find examples of both Division and Composition in the practice so common in debate of 'taking to pieces' the arguments by which any theory or proposed course of action is justified. A person would be guilty of Division if he should argue that, because a complex theory is not completely proved, none of the arguments by which it is supported have any value. It is, however, perhaps more common to fall into the fallacy of Composition in combating the arguments of an opponent. Some measure, for example, is proposed to which a person finds himself in opposition. It is usually easy to analyze the different arguments which have been advanced in support of the measure, and to show that no single one of these *taken*

by itself is sufficient to justify the change. The conclusion may then be drawn with a fine show of logic that *all* the reasons advanced have been insufficient. This, of course, is to neglect the cumulative effect of the arguments; it is to assume that what is true of 'all,' taken distributively, is also true of 'all' when taken in conjunction.

(4) It is often difficult to distinguish the various forms of the fallacy of **Accident** from Composition and Division. We have seen that the latter rest upon a confusion between whole and part; or, as we have already expressed it, on an equivocation between the distributive and collective use of terms. The fallacies of Accident are also due to Equivocation. But in this case the confusion is between essential properties and accidents, between what is true of a thing in its real nature, as expressed by its logical definition, and what is true of it only under some peculiar or accidental circumstance.

There are two forms of this argument which are usually recognized: (*a*) The *Direct* or *Simple* Fallacy of Accident, which consists in arguing that what is true of a thing generally is also true of it under some accidental or peculiar circumstance. The old logicians expressed this in the formula, *a dicto simpliciter ad dictum secundum quid.* The second form is (*b*) the *Converse* Fallacy of Accident, which consists in arguing that what is true of a thing under some condition or accident, can be asserted of it simply, or in its essential nature. The formula for this is, *a dicto secundum quid ad dictum simpliciter.*

It would be an illustration of the direct fallacy to

reason, that because man is a rational being, therefore a drunken man or an angry man will be guided by reason. Similarly, we should commit this fallacy if we were to argue that because beefsteak is wholesome food, it would be good for a person suffering with fever or dyspepsia; or to conclude from the principle that it is right to relieve the suffering of others, that we ought to give money to beggars.

It would be a case of the converse fallacy to argue, that because spirituous liquors are of value in certain cases of disease, they must therefore be beneficial to a person who is well. We should also be guilty of the same fallacy if we should conclude that it is right to deceive others, from the fact that it is sometimes necessary to keep the truth from a person who is sick, or to deceive an enemy in time of war.

The fallacies of Accident, like all the fallacies of Equivocation, are largely the result of a loose and careless use of language. By qualifying our terms so as to state the exact circumstances involved, they may easily be detected and avoided.

(*B*) *Fallacies of Presumption.* — The fallacies of this group are the result of presumption or assumption on the part of the person making the argument. It is possible (1) to assume the point to be proved, either in the premises of an argument, or in a question (*Petitio Principii*, and Complex Question); or (2) to assume without warrant that a certain conclusion follows from premises which have been stated (*Non Sequitur*); or (3) that the conclusion obtained proves the point at issue (Irrelevant Conclusion).

(1) *Petitio Principii*, or 'Begging the Question,' is a form of argument which assumes the conclusion to be proved. This may be done in either of two ways. (1) We may postulate the fact which we wish to prove, or its equivalent under another name. Thus, for example, we might argue that an act is morally wrong because it is opposed to sound ethical principles. 'The soul is immortal because it is a simple and indecomposable substance,' may be regarded as another example of this assumption. But (2) the question may be begged by making a general assumption covering the particular point in dispute. Thus, if the advisability of legislation regulating the hours of labor in a mine or factory were under discussion, the question-begging proposition, 'all legislation which interferes with the right of free contract is bad,' might be propounded as a settlement of the whole question.

A special form of this fallacy results when each of two propositions is used in turn to prove the truth of the other. This is known as 'reasoning in a circle,' or *circulus in probando*. This method of reasoning is often adopted when the premise, which has been employed to prove the first conclusion, is challenged. 'I should not do this act, because it is wrong.' 'But how do you know that the act is wrong?' 'Why, because I know that I should not do it.'

It is always necessary, then, to see that the conclusion has not been assumed in the premises. But, since the conclusion always follows *from* the premises, we may say in one sense that the conclusion is always thus assumed. It is, therefore, easy to charge an opponent

unjustly with begging the question. De Morgan in his work on Fallacies, says: "There is an opponent fallacy to the *Petitio Principii* which, I suspect, is of more frequent occurrence: it is the habit of many to treat an advanced proposition as a begging of the question the moment they see that, if established, it would establish the question." All argument must, of course, start from premises to which both parties assent. But candour and fairness forbid us to charge an opponent with *Petitio* because the results of his premises are unwelcome. It was Charles Lamb who humorously remarked that he would not grant that two and two are four until he knew what use was to be made of the admission.

(2) **The Complex Question** is an interrogative form of *Petitio.* It is not really a simple interrogation, but is founded upon an assumption. Examples may be found in popular pleasantries, such as, 'Have you given up your drinking habits?' 'Do the people in your part of the country still carry revolvers?' Disjunctive questions, too, always contain an assumption of this kind: 'Is this an oak or an ash?' 'Does he live in Boston or New York?' The 'leading questions' which lawyers frequently use in examining witnesses, but which are always objected to by the opposing counsel, are usually of this character. Further instances may perhaps be found in the demand for explanation of facts which are either false, or not fully substantiated; as, *e.g.* 'Why does a fish when dead weigh more than when alive?' 'What is the explanation of mind-reading?'

(3) **The Irrelevant Conclusion**, or *Ignoratio Elenchi*,

consists in substituting for the conclusion to be proved some other proposition more or less nearly related to it This fallacy may be the result of an involuntary confusion on the part of the person employing it, or it may be consciously adopted as a controversial stratagem to deceive an opponent or an audience. When used in this latter way, it is usually intended to conceal the weakness of a position by diverting attention from the real point at issue. This is, indeed, a favourite device of those who have to support a weak case. A counsel for the defence in a law-suit is said to have handed to the barrister presenting the case the brief marked, 'No case; abuse the plaintiff's attorney.' To answer a charge or accusation by declaring that the person bringing the charge is guilty of as bad, or even worse, things, — what is sometimes called the *tu quoque* form of argument — is also an example of this fallacy.

Apart from such wilful perversions or confusions, many unintentional instances of this fallacy occur. In controversial writing, it is very natural to assume that a proposition which has some points of connection with the conclusion to be established, is 'essentially the same thing,' or 'practically the same, as the thesis maintained.' Thus one might take the fact that a great many people are not regular church-goers, as a proof of the proposition that religion and morality are dying out in the country. Many of the arguments brought against scientific and philosophical theories belong to this class. Mill cites the arguments which have been urged against the Malthusian doctrine of population, and Berkeley's theory of matter. We may quote the

passage referring to the former: "Malthus has been supposed to be refuted if it could be shown that in some countries or ages population has been nearly stationary, as if he had asserted that population always increases in a given ratio, or had not expressly declared that it increases only, in so far as it is not restrained by prudence, or kept down by disease. Or, perhaps, a collection of facts is produced to prove that in some one country with a dense population the people are better off than they are in another country with a thin one, or that the people have become better off and more numerous at the same time; as if the assertion were that a dense population could not possibly be well off." [1]

There are several cases or forms of Irrelevant Conclusion to which special names have been given, and which it is important to consider separately. When an argument bears upon the real point at issue, it is called *argumentum ad rem*. But, on the other hand, there are the following special ways of obscuring the issue: — *argumentum ad hominem*, *argumentum ad populum*, *argumentum ad ignorantiam*, and *argumentum ad verecundiam*.

The *argumentum ad hominem* is an appeal to the character, principles, or former profession of the person against whom it is directed. It has reference to a person or persons, not to the real matter under discussion. In order to confuse an opponent, and discredit him with the audience, one may show that his character is bad, or that the views which he is now maintaining

[1] *Logic*, Bk. V. Ch. VII. § 3.

are inconsistent with his former professions and practice. Or the argument may be used with the hope of persuading the opponent himself. We then try to convince him that the position which he maintains is inconsistent with some other view which he has previously professed, or with the principles of some sect or party which he has approved. Or we may appeal to his interests by showing him that the action proposed will affect injuriously some cause in which he is concerned, or will benefit some rival sect or party. In all of these cases the real point at issue is, of course, evaded.

The *argumentum ad populum* is an argument addressed to the feelings, passions, and prejudices of people rather than an unbiassed discussion addressed to the intellect.

The *argumentum ad ignorantiam* is an attempt to gain support for some position by dwelling upon the impossibility of proving the opposite. Thus we cannot prove affirmatively that spirits do not revisit the earth, or send messages to former friends through 'mediums.' Now it is not unusual to find ignorance on this subject advanced as a positive ground of conviction. The argument seems to be: —

> It is not impossible that this is so,
> What is not impossible is possible,
> Therefore it is possible that this is so.

The fallacy arises when we confuse what is only abstractly possible — *i.e.*, what we cannot prove to be impossible — with what is really possible, *i.e.*, with what we have *some* positive grounds for believing in, though those grounds are not sufficient to produce conviction.

The *argumentum ad verecundiam* is an appeal to the reverence which most people feel for a great name. This method of reasoning attempts to settle a question by referring to the opinion of some acknowledged authority, without any consideration of the arguments which are advanced for or against the position. It is, of course, right to attach much importance to the views of great men, but we must not suppose that their opinion amounts to proof, or forbids us to consider the matter for ourselves.

There is, however, a more common, though still less justifiable, form of the argument from authority. A man who is distinguished for his knowledge and attainments in some particular field, is often quoted as an authority upon questions with which he has no special acquaintance. The prestige of a great name is thus irrelevantly invoked when no significance properly attaches to it. Thus, for example, a successful general is supposed to speak with authority upon problems of statescraft, and the opinions of prominent clergymen are quoted regarding the latest scientific theories.

(4) The **fallacy of non sequitur**, or *the fallacy of the consequent*, occurs when the conclusion does not really follow from the premises by which it is supposed to be supported. The following example may serve as an illustration: —

> Pennsylvania contains rich coal and iron mines,
> Pennsylvania has no sea-coast,
>
> Therefore the battle of Gettysburg was fought in that state.

This argument, of course, is thoroughly inconsequent,

and would deceive no one. But when the conclusion repeats some words or phrases from the premises, we are likely, when not paying close attention, to be imposed upon by the mere form of the argument. We notice the premises, and remark that the person using the argument advances boldly through 'therefore' to his conclusion. And if this conclusion appears to be related to the premises, and sounds reasonable, the argument is likely to be accepted. The following example will illustrate this: —

Every one desires happiness, and virtuous people are happy,
Therefore every one desires to be virtuous.

What is known as the **False Cause** (*non causa pro causa; post hoc ergo propter hoc*) is the inductive fallacy corresponding to the *non sequitur*. In this we assume that one thing is the cause of another merely because we have known them to happen together a number of times. The causal relation is assumed without any analysis or examination, on the ground of some chance coincidence. Thus a change in the weather may be attributed to the moon, or the prosperity of the country to its laws requiring Sunday observance (cf. pp. 255 f.).

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PART II.—INDUCTIVE METHODS

CHAPTER XIII

THE PROBLEM OF INDUCTION. OBSERVATION AND EXPLANATION

§ 47. **The Problem of Induction.** — In Part I. we have outlined the general nature of the syllogism, and have shown what conditions must be fulfilled in order to derive valid conclusions from given premises. But the syllogism does not represent completely all of our ways of thinking. We do not always find premises which every one accepts ready to our hand. The propositions which serve as the premises of syllogisms are themselves the result of the activity of thought. It requires thinking to arrive at such simple propositions as, 'all men are mortal,' 'water is composed of hydrogen and oxygen.' Facts of this kind are of course learned through experience, but they none the less require thought for their discovery. Sense-perception without thought could give us only a chaos of unordered impressions which would have no meaning and no significance. It is important, then, to understand how our intelligence proceeds to discover the real nature of things, and the laws according to which they operate. Thinking is the means by which we interpret nature, and to show how this is to

be accomplished was the purpose of Bacon's *Novum Organum*. The problem is the discovery of the real nature of things, and their relations with one another. The assumption of all knowledge, as we have already seen (§ 9, cf. also §§ 79, 80), is that there is a permanent constitution of things which secures uniform ways of acting. The procedure by means of which intelligence discovers the permanent laws of things is usually known as Induction. We shall have to study this kind of thinking in this and the following chapters. The general problem may perhaps be stated in this way: What are the methods which inductive thinking employs, in order to pass from the chaotic and unordered form in which the senses present our experience, to a perception of the order and law in things that is required by real knowledge or science?

Before we attempt to answer this question, however, there are several remarks to be made which will, I hope, throw further light upon the nature of our undertaking. In the first place, it is to be noticed that we have spoken in the preceding paragraph of the methods of inductive thinking. Now, as we shall show more fully in § 88, there is no essential difference between the *results* of an inductive and a deductive inference. The purpose of an inference is always the same: namely, to exhibit the relation and connection of particular facts or events in virtue of some universal law or principle. In deductive thinking, such a law is known, or provisionally assumed as known, and the problem is to show its application to the facts with which we are dealing. In induction, on the other hand,

the starting-point must be the particular facts, and the task which thought has to perform is to discover the general law of their connection. Both deduction and induction play an important part in the work of building up knowledge. But the various sciences must start with particular facts learned through experience. The mind has not before experience any store of general principles or innate truths which might serve as the starting-point of knowledge (cf. § 76). It must fall back, therefore, upon the particular facts and events learned through perception. This 'elementary knowledge,' as has been already pointed out, does not pass over in a ready-made form into the mind, but is itself the result of thinking or judging. However, before any one deliberately and consciously undertakes to discover new truth, to understand the world, he is already in possession of a store of such perceptive judgments. These constitute the beginnings of knowledge, and serve as the starting-point for scientific explanation. The knowledge of laws and general principles comes later, and is derived from a study of the particular facts. It is clear, then, that the procedure of all the sciences must be inductive, at least in the beginning. The various sciences are occupied, each in its particular field, with an attempt to reduce to order and unity facts, which at first sight appear to be lawless and disconnected. And it is true to say that in this undertaking the general procedure is inductive. But it will also appear that in performing this task thought does not always proceed in strictly inductive fashion. Our thought uses every means which will help it to its desired end. It is often

able, after pushing its inquiries a little way, to discover some general principle, or to guess what the law of connection must be. When this is possible, it is found profitable to proceed deductively, and to show what results necessarily follow from the truth of such a general law. Of course, it is always essential to verify results obtained in this deductive way, by comparing them with the actual facts. But in general, the best results are obtained when induction and deduction go hand in hand. We shall expect to find, then, that the so-called inductive methods sometimes include steps which are really deductive in nature.

It is to be noticed, further, that in dealing with the nature of the inductive methods, we are not laying down rules which thought must follow. We are not attempting, that is, to *prescribe* to thinking its mode of procedure. To do so would be quite futile. It is impossible, as we have already seen (§ 3), for logic to lay down any *a priori* rules. Its task is rather to point out the methods by which success has been already won in the various fields of knowledge. Logic does not attempt to *invent* any methods of scientific procedure, but it undertakes to *describe* the road by which truth has already been gained. The scientific inquirer is interested primarily in the results of his thinking: he is usually not interested in tracing the various steps through which his thought has passed, and the methods employed in reaching the goal. Oftentimes he would be unable to give any such description even if he tried to do so. Logic, however, takes the procedure of the thinking process for its subject-matter. It undertakes to make thought conscious of its own nature, of the goal at which it aims, and the methods which are employed in the attainment of this goal. The comparative value of these methods, too, must be decided by the actual character of the results which they have yielded. One method is to be regarded as better than another when it gives us knowledge which is universally acknowledged to be more complete and satisfactory

than that afforded by the other. For logical methods, like everything else, must be known and judged by their fruits.

Again, it must be remembered that complete scientific explanation, which we found to be the type of perfect knowledge, is not attained at a single stroke. Scientific inquiry may have various purposes. It is often limited to an attempt to gain a knowledge of the quantitative relations of things, or of the way in which they are connected as antecedents and consequents. In some cases, too, the conclusions reached are only more or less probable, and require further confirmation through the use of other methods. It follows, then, that the various scientific methods which we shall have to describe are not to be regarded as self-sufficient and independent ways of reaching truth, but rather as mutually helpful and complementary. For example, the work done by thought in dealing with the quantitative aspect of things, and the conclusions which it reaches through analogical inference, are necessary steps in the progress toward complete and satisfactory explanations of the nature of things. We shall find it necessary, therefore, to keep in mind in our investigation this relation of the various methods to one another. For our purpose, we may perhaps classify the various scientific methods as those of Observation and Explanation, the latter including Analogy and Complete Scientific Explanation.

§ 48. **Observation.** — We may include under this heading, *Simple Enumeration, Statistical Methods, and Methods of determining Causal Connection.* Before describing these processes in detail, however, it is neces-

sary to make clear what is implied in the nature of scientific observation, and what are the results aimed at by the methods which it employs. It is customary to say that **Observation** has to determine the nature and order of the particular facts presented by our experience, and that after this has been done, there still remains the task of furnishing the theory, or **Explanation** of the facts. This distinction, though not absolute, affords a convenient principle of division in treating of the inductive methods. We may say that it is observation which enables us to discover the nature of particular facts, and to determine the order of their connection. Accurate observation is thus a first and necessary step in the work of reducing our experience to systematic form. We have already seen how emphatically and eloquently this doctrine was proclaimed by Bacon in the *Novum Organum*.

It is important, however, to remember that scientific observation itself involves intellectual activity. To observe — at least in the sense in which the word is used in scientific procedure — requires something more than the passive reception of impressions of sense in the order in which they come to us. Without some activity on the part of mind, it would be impossible to obtain even the imperfect and fragmentary knowledge of everyday life. But accurate observation is one of the means which science employs to render this knowledge more complete and satisfactory; and when observation thus becomes an exact and conscious instrument, it involves, to even a greater extent than in ordinary life, intellectual activities like judgment and inference. It is because this is true, because scientific observation

demands the constant exercise of thought, in selecting and comparing the various elements in the material with which it deals, that it affords such excellent intellectual discipline. The observational sciences do not merely train the sense-organs; the discipline which they afford is mental as well as physiological, and it is, of course, true that mental training can only be gained through the exercise of mental activity.

It is quite true that it is of the utmost importance to distinguish between a fact, and further inferences from the fact. As will be pointed out in the chapter on Inductive Fallacies, errors very frequently arise from confusing facts and inferences. The point which is emphasized in the previous paragraph, however, is that it requires a certain amount of *thinking* in order to get a fact at all. Facts do not pass over ready-made into the mind. Simply to stare at things does not give us knowledge; unless our mind reacts, judges, thinks, we are not a bit the wiser for staring. To observe well, it is necessary to be more or less definitely conscious of what one is looking for, to direct one's attention towards some particular field or object; and to do this implies selection among the multitude of impressions and objects of which we are conscious. Moreover, scientific observation requires analysis and discrimination. It is not unusual, in text-books on logic, to symbolize the various facts learned through observation by means of letters, *a*, *b*, *c*, etc., and to take it for granted that they are given in our experience as distinct and separate phenomena; but, as we have just seen, judgments of analysis and discrimination are necessary to separate out the so-called 'phenomena' from the mass or tangle of experience in which they were originally given. Again, to determine the nature of a fact through observation, it is essential to note carefully how it differs from other facts with which it is likely to be confused, and also, to some extent, what relations and resemblances it has. But such knowledge presupposes that thought has already been at work in forming judgments of comparison.

It may seem strange at first sight that the determination of the causal order and connection of phenomena should be regarded as belonging to Observation rather than to Explanation. To discover the causes of things is, indeed, a very essential step in the process of explanation; but, as will appear more fully hereafter, the distinction between observation and explanation is not an absolute one. The process of knowledge is essentially the same from beginning to end. The determination of the nature and order of phenomena is a long step towards rendering them comprehensible. If the distinction between observation and explanation as methods of scientific procedure is to be made, it seems right to assign to observation the task of determining what phenomena are invariably conjoined as antecedents and consequents. Experience presents to us a variety of objects simultaneously or in rapid succession, but in many cases such conjunction is merely temporary and accidental. The problem which scientific observation has here to determine is the discovery of what particular phenomena are *necessarily* connected, what are the real antecedents and consequents in the case. 'The sun was very hot this morning, and a picnic party went on the lake, and this afternoon there is a severe thunderstorm.' These events (and many others) are conjoined temporally. Is there also a real connection between any of them, or is their concurrence merely accidental? This is the question which must be answered by the methods of determining causal connection. Of course merely passive observation will not suffice to obtain an answer. The relation of antecedent and consequent is

not given, but has to be made out by the help of analysis and inference. But, since the point to be determined has reference to the nature and order of a set of facts which can be observed, the methods employed may well be included under Observation.

A distinction is sometimes made between observation and experiment. In observation, it is said, the mind simply *finds* its results presented to it in nature, while in experiment the answer to a question is obtained by actively controlling and arranging the circumstances at will. There are, no doubt, some grounds for this distinction, though it is not true that the mind is passive in the one case, and active in the other. Even in observation, as we have seen, knowledge always arises through active analysis and comparison of the impressions received through sense. The difference is rather this: In observing, where experiment is impossible, one must wait for events to occur, and must take them in the order in which they are presented in the natural series. But, where experiment is employed, we have control of the conditions, and can produce the phenomena to be investigated in any order, and as often as we choose. In experiment, as Bacon says, we can put definite questions to nature, and compel her to answer. This is, of course, an immense advantage. In some of the sciences, however — geology and astronomy for example — it is not possible thus to control the conditions: one must wait and observe the results of nature's experiments. Physics and chemistry are the experimental sciences *par excellence;* and, in general, we may say that a science always makes

more rapid progress when it is found possible to call experiment to the aid of observation. It is not possible to conceive how physics and chemistry could have reached their present state of perfection without the assistance of experiment. Indeed, the almost total neglect of experiment by the Greek and mediæval scholars must be regarded as one of the chief reasons why the physical sciences made so little progress during those centuries. Dr. Fowler states in the following passage some of the main advantages to be derived from experiment: —

"To be able to vary the circumstances as we choose, to produce the phenomenon under investigation in the precise degree which is most convenient to us, and as frequently as we wish, to combine it with other phenomena or to isolate it altogether, are such obvious advantages that it is not necessary to insist upon them. Without the aid of artificial experiment it would have been impossible, for instance, to ascertain the laws of falling bodies. To disprove the old theory that bodies fall in times inversely proportioned to their weight, it was necessary to try the experiment; to be able to affirm with certainty that all bodies, if moving in a non-resisting medium, would fall to the earth through equal vertical spaces in equal times, it was essential to possess the means of removing altogether the resisting medium by some such contrivance as that of the air-pump. . . . Even when observation alone reveals to us a fact of nature, experiment is often necessary in order to give precision to our knowledge. That the metals are fusible, and that some are fusible at a lower temperature than others, is a fact which we can conceive to have been obtruded upon man's observation, but the precise temperature at which each metal begins to change the solid for the liquid condition could be learned only by artificial experiment."[1]

It is important, then, to recognize the services which

[1] Fowler, *Inductive Logic*, p. 41 f.

experiment renders in helping us to understand the facts with which the various sciences deal. But it is not necessary to distinguish experiment from observation as if it were a separate and independent mode of investigation. We should rather say that observation, in the sense in which we have used the word, employs experiment wherever practicable as an indispensable auxiliary. The methods of observation, then, which have still to be described, will in many cases call for the employment of experiments. Indeed, it will be seen that some of these are essentially methods of experimentation.

§ 49. **Explanation.** — We have already seen that the distinction between observation and explanation is not an absolute one. The task which thought has to perform — the task which is undertaken by science — is to reduce the isolated and chaotic experiences of ordinary life to order and system. And it is important to remember that all the various methods employed contribute directly towards that result. It has, however, seemed possible to divide this undertaking into two main divisions. Observation, it was said, seeks to discover the exact nature of the facts to be dealt with, and also to determine the ways in which they are necessarily and invariably connected. But, when this has been accomplished, we have not by any means reached an end of the matter. The desire for knowledge is not satisfied with a mere statement of facts, or with the information that certain phenomena always occur in a fixed order as antecedents and consequents. Complete knowledge demands an *explanation* of the facts as thus determined

by the methods of observation. '*Why*,' we ask, 'should *a* always precede *b*?' '*Why* should dew be deposited under such and such conditions, or water rise thirty-two feet in a pump?' Science, we feel, should do more than describe the facts; it should offer an explanation of them as well. To explain events, however, is to furnish reasons for them. The scientist is not content to know merely *that* such and such phenomena exist, and occur in conjunction with each other, but he attempts to discover *why* this is so. His knowledge is not confined to the 'what,' but includes the 'why.' It is, of course, true that a large part of scientific work is occupied with an attempt to determine precisely and accurately the nature of facts. Until the facts are thus scientifically determined attempts at explanation are usually quite futile. But after this has been accomplished, it is still necessary to show reasons why the phenomena with which we are dealing have the precise character which they are found to possess, and why they should occur in the invariable order in which they are observed. Explanation, in other words, *completes* the knowledge obtained through observation. It does further intellectual work on the results given by the latter process. Explanation, itself, has various degrees of completeness; it may be more or less satisfactory. When we come to treat Analogy, for example, we shall find that it affords a kind of explanation, though not one of an entirely satisfactory type. In general, however, we may say that explanation goes beyond the particular facts, and seizes upon general principles or laws to which the facts are referred. And it is only when one knows the general law

or principle involved in the case, that one can be said really to understand the particular facts.

It is usually said that where we know merely the nature of phenomena, and their connection, without being able to explain these facts, our knowledge is *empirical*. Thus, I may know that an explosion follows the contact of a lighted match with gunpowder, or that a storm follows when there is a circle around the moon, without being able to explain in any way why these facts are connected. On the other hand, if we can connect events by showing the general principle involved, we say that our knowledge is really scientific. It is important to notice, however, that empirical knowledge is simply in a less advanced stage than the scientific knowledge which has succeeded in gaining an insight into the general law. Empirical knowledge leaves a problem which intelligence has still to solve. It is, of course, truc that a large part of every one's knowledge is empirical in character. We all know many things which we cannot explain. In all the sciences, too, phenomena are met with which seem to defy all attempts at explanation. Indeed, some of the sciences can scarcely be said to have passed the empirical stage. The science of medicine, for example, has hardly yet reached any knowledge of general principles. The physician knows, that is, as a result of actual experiment, that such and such drugs produce such and such effects. But he knows almost nothing of the means by which this result is achieved, and is therefore unable to go beyond the fact itself. In this respect, he is very little better off than the ordinary man, who knows that if he eats certain kinds of food he will be ill, or if he drinks strong liquors in excess he will become intoxicated.

CHAPTER XIV

METHODS OF OBSERVATION. — ENUMERATION AND STATISTICS

§ 50. **Enumeration or Simple Counting.** — We shall begin the account of the scientific methods with Enumeration. To count the objects which we observe, and to distinguish and number their parts, is one of the first and most essential operations of thought. It is of course true that qualitative distinctions precede quantitative. The child learns to distinguish things by some qualitative mark, such as 'black' or 'hot,' before he is able to count them (cf. § 82). But we may say, nevertheless, that the qualities of things are known, in a general way at least, before scientific procedure begins. The determination of quantity, on the other hand, seems to demand a more conscious effort on the part of the mind. We learn, that is, to distinguish the general qualities of things without effort, but, in order to obtain quantitative knowledge, it is necessary to set ourselves deliberately to work. We may, therefore, take Enumeration, or Simple Counting, which is perhaps the easiest kind of quantitative determination, as our starting-point in dealing with the Inductive Methods.

A considerable step in advance, in the task of reducing the world of our experience to order and unity, is taken when we begin to count, *i.e.*, to group together

things of the same kind, and to register their number. Thus Enumeration is, to some extent, also a process of classification. What is counted is always a collective whole, the units of which are either all of the same kind, or else belong to a limited number of different classes. Thus one might determine by Enumeration the number of sheep in a flock, taking each individual as belonging to the same general class, 'sheep'; or the analysis might be pushed further so as to give as a result the number of white and of black sheep separately. The purpose for which the enumeration is undertaken always determines the length to which the process of analysis and distinction is carried. For example, if the object of a census enumeration were simply to determine the number of inhabitants in a country, it would not be necessary to make any distinctions, but each person would count as one. But where, as is often the case, the aim is not simply to count the sum-total, but also to determine the relative numbers belonging to various classes, analysis has to be pushed further. In such cases, we might count the number belonging to each sex, the native-born, and those of foreign birth, those below, and those above any given age, etc.

It will be noticed that the process of enumeration takes account of each individual instance. And the judgment which sums up the process puts the result in a numerical form. 'There are twenty-five thousand inhabitants in this town, five thousand of whom are of foreign birth.' In cases where the examination of particular instances has been exhaustive, the result may be stated in the form of a universal proposition. Thus,

after examining the calendar of each of the months separately, we might say: 'All of the months contain less than thirty-two days.' Or, after measuring each individual in a company, the assertion might be made: 'No one in this company is more than six feet tall.' Cases of this kind, where a general assertion is made after an examination of all the individuals concerned, are termed by Jevons, instances of *Perfect Induction.* "An Induction, that is an act of Inductive reasoning, is called Perfect, when all the possible cases or instances to which the conclusion can refer, have been examined and enumerated in the premises."[1] On the other hand, where, as usually happens, it is impossible to examine all the cases, the inductive process is regarded as *Imperfect* by the same writer, and the conclusion expressed in the general law as only probable. The assertion that all the months of the year contain less than thirty-two days, is derived from Perfect Induction, and is absolutely certain, but the proposition that 'all men are mortal,' is derived from Imperfect Induction, and there is no certainty, but only a probability that all future cases will agree with those which we have already experienced.

This distinction, however, seems to be founded on a mistaken view of the nature of inductive reasoning. It assumes that it is the business of induction to count instances. When the examination and enumeration is exhaustive, the results can, of course, be summed up in a general proposition which is absolutely certain. But

[1] Jevons, *Elementary Lessons in Logic*, pp. 212–213.

where the counting is incomplete, where all the possible cases cannot be examined, the conclusion is regarded as uncertain. Now, this could be accepted as an account of induction, only if it were maintained that this process aims merely at a summation of particular instances. We have already seen, however, that the real object of inductive inference is to discover the general law or principle which runs through and connects a number of particular instances. It is, of course, true that we shall be more likely to obtain a correct insight into the nature of the law from an examination of a large number of cases than from that of a small number. But the discovery of the principle, and not the number of instances, is the main point. If the purpose of the induction, the discovery of the universal principle, can be adequately attained, one case is as good as a hundred (cf. § 88).

The truth seems rather to be that enumeration is merely the beginning, rather than the end of the inductive process. It gives us important information regarding particular instances and individuals. But in itself it is not sufficient to bring to light the general law that explains why the particular objects enumerated are connected together, or act as they do. Enumeration plays a part as a method of observation, but it affords no real explanation of the particular facts with which it deals. Even where all the possible cases are examined, it cannot rightly be called Perfect Induction, for the goal of Induction is explanation by means of a general principle. The requirements of inductive science are not completely fulfilled, for example, when an examination of Mercury, Venus, Mars, and all the other known planets yields the proposition: 'All the planets revolve around the sun in elliptical orbits.' The 'all' in this proposition denotes simply an aggregate of individuals. It is merely an expression of fact. The reasons necessary

to explain the fact are not reached by enumeration; in order to obtain them it is necessary that further work shall be done by thinking, and that the process of induction shall be carried further.

The conclusion which we reach, then, is that no process of enumeration has any claim to the title of Perfect Induction. Enumeration is the beginning, rather than the end of the inductive procedure. Nevertheless, it is exceedingly useful as a preliminary step and preparation for scientific explanation. The number of stamens and pistils which a plant contains, or the number of tympanic bones possessed by an animal, is often of the greatest service in classification. And classification, although it is by no means the end of scientific investigation, is in many of the sciences a most essential and important step towards it. The task of explaining the infinite variety of natural objects would be a hopeless one, if it were not possible to discover similarities of structure, in virtue of which things can be grouped together in classes. To this, enumeration in a very great degree contributes.

§ 51. **Statistics and Statistical Methods.** — Statistical methods depend upon enumeration. They aim at making the process of counting as exact and precise as possible. Modern science has come to understand that its first task must be to become acquainted, as completely as possible, with the nature of the facts presented to it by experience. And, for this purpose, the careful classification and precise enumeration of particulars afforded by statistics, is often of the greatest importance. "The extent to which the statistical method prevails, and

everything is counted," says Professor Sigwart, "is another instance of the fundamental difference between ancient and modern science."[1] It would, of course, be impossible to enter here into a full description of the methods employed by statistical science. The methodology of every science must be learned by actual practice within the particular field. What we are interested in from a logical point of view is the purpose which statistical investigation seeks to fulfil, and the part which it plays in rendering our knowledge exact and systematic.

We notice, in the first place, that the class of facts to which statistics are applied has two main characteristics: the subject dealt with is always complex, and capable of division into a number of individual parts or units; and, secondly, it is also of such a nature that the underlying law or principle of the phenomena to be investigated cannot be directly discovered. Thus, we employ statistics to determine the death-rate of any country or community, or the ratio between the number of male and of female births. It is clear that it is impossible to make use of experiment when we are dealing with facts of this kind, because the conditions are not under our control. If it were possible, for example, to determine exhaustively the general laws according to which the various meteorological changes are coördinated with their conditions, we should not trouble ourselves to count and register the separate instances of changes in the weather. Nor, if we knew exactly the general condi-

[1] *Logic* (Eng. trans.), Vol. I., p. 286.

tions under which any given human organism in contact with its environment would cease to exist, should we count the individual cases of death. "In proportion as we are unable to reduce the particular event to rules and laws, the numeration of particular objects becomes the only means of obtaining comprehensive propositions about that which is, for our knowledge, fortuitous; as soon as the laws are found, statistical numeration ceases to be of interest. There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur, and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunder-storms and hail-storms occur at a given place, or within a given district, how many persons die, and how many bushels of fruit a given area produces, because we are not in a position to calculate these events from their conditions."[1]

In cases like those mentioned above, where we are as yet unable to determine the general laws which are at work, we call to our aid statistical enumeration. There are two main advantages to be derived from the employment of this method. In the first place, it contributes directly towards a clear and comprehensive grasp of the facts. Instead of the vague impression derived from ordinary observation, statistics enable us to state definitely the proportion of fine and rainy days during the year. Statistical enumeration is thus one

[1] Sigwart, *Logic* (Eng. trans.), Vol. II., p. 483.

of the most important means of rendering observation exact and trustworthy, and of summing up its results in a convenient and readily intelligible form. It is of the utmost importance when dealing with complex groups of phenomena, to have a clear and comprehensive view of the facts of the case. Thus, when trying to understand the nature of society, it is necessary to determine accurately by means of statistics, such facts as the number of male and of female births, the death-rate, the proportion of marriages, the age of marriage, etc. But, in the second place, statistics often serve to reveal quantitative correspondences or uniformities between two groups of phenomena, and thus suggest that some causal connection exists between them. It is found, for example, that the number of births in any given country varies inversely as the price of food during the previous year. Now this fact at once suggests the existence of certain physiological and psychological laws which may serve to bring these facts into causal relation. In many cases, such correspondences serve only to confirm our expectation of the presence of a causal law, which is based on other grounds. Thus we should naturally expect that there would be a relatively greater number of cases of fever in a town which had an insufficient water supply, or an antiquated system of sewerage, than in a town where these matters were properly provided for; and statistics might bear out our conclusions. In general, however, it may be said that causal laws are suggested, not by corresponding uniformities, but by corresponding variations, as shown by the statistics of different sets of facts. So long as the death-

rate, for example, shows a constant ratio to the population, no causal inference is suggested; but if the annual number of deaths increases or decreases considerably, we are led to look for some variation from the normal in some coincident group of phenomena. And if it is found that the variation in the death-rate has been accompanied by unusually favourable or unfavourable conditions of weather, the presence or absence of epidemics, or any similar circumstances, there will be at least a *presumption* that a causal relation exists between these two sets of events. From a certain likeness or quantitative resemblance between the variations of two distinct classes of phenomena, we are led to the hypothesis of their causal connection.

Statistical enumeration is frequently employed to determine the *average* of a large number of instances of a particular kind. This is obtained by dividing the sum of the given numbers by the number of individuals of which account is taken. In this way a *general average* is reached which does not necessarily correspond exactly with the character of any individual of the group. It represents a purely imaginary conception, which omits individual differences and presents in an abbreviated form the general character of a whole class or group. In this way, by the determination of the average, it becomes easier to compare complex groups with one another. Thus, when the average height of Frenchmen and Englishmen has been determined, comparison is at once made possible. For the mean or average of a number of individuals, or set of instances, however, we can infer nothing regarding the character of any particular individual, or of any particular instance. What *is* determined by the method of averages is the general nature of the group, as represented by the average or typical individual. But this method does not enable us to infer anything regarding the character of any member of the group, A, or B. When exact statistics are obtainable, however,

it is possible to show what the *probabilities* are in reference to any particular case, so long as the peculiar circumstances which belong to each instance are not considered, and each case is reckoned simply as one unit of the group. This is, of course, the principle employed by the method of mathematical probabilities. It will be sufficient here to indicate the general method of procedure in such cases.

§ 52. **The Calculation of Chances.** — There is, of course, no such thing as 'chance,' regarded as a power which controls and governs events. When we speak of something happening 'by chance,' or of some occurrence as 'probable,' we are expressing merely a deficiency in our own knowledge. "There is no doubt in lightning as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach but infinite knowledge would account for its lying there; and the course of every falling leaf is guided by the same principles of mechanics as rule the motions of the heavenly bodies."[1] To assert that anything happens by chance, then, is simply to confess our ignorance of the causes which are operative.

It is clear that we are in this position regarding many of the ordinary events which belong to the future. Because of my ignorance of the causes at work, I can only say, 'It *may* rain to-morrow.' It is impossible to tell upon which side a penny will fall at any particular throw, or what card may be drawn from a pack. But in cases like these, we have to accept, for lack of anything better, a numerical statement of the chances for any particular event. Thus we know that, since there

[1] Jevons, *The Principles of Science*, Vol. I., p. 225.

are only two sides upon which a penny can fall, the chances of throwing heads in any trial is $\frac{1}{2}$. Similarly, there are four chances out of fifty-two of drawing an ace from a pack of cards. The chance of obtaining an ace by any draw is therefore $\frac{4}{52}=\frac{1}{13}$. These figures express the mathematical chances. Experience of a limited number of instances may, however, sometimes appear to show a lack of harmony between the mathematical and the actual chances. But in proportion as the number of trials is increased, the result is found to approximate more and more nearly to the mathematical expectation. In twenty throws of a penny or a die, we should not be surprised to find that the result differed from the fraction expressing the mathematical chances. But this discrepancy would tend to disappear as the number of cases was increased. Jevons illustrated this by actual trial, using a number of coins at a time. Out of a total of 20,480 throws, he obtained a result of 10,353 heads. On the result of the experiment he remarks: "The coincidence with theory is pretty close, but considering the large number of throws there is some reason to suspect a tendency in favor of heads."[1]

Apart from the simple and somewhat artificial cases where we are concerned with coins and dice, etc., it is impossible to determine with mathematical precision the chances for or against any event. In cases where the whole series of possibilities does not lie before us, we have to base our calculations for the future on what is known regarding the frequency with which the events

[1] Jevons, *loc. cit.* Vol. I., p. 230.

under consideration have occurred in the past. Now the results of the last paragraph make it clear that it is of the utmost importance that the statistics, which are taken as the basis, shall be as full and comprehensive as possible. It is evident, for example, that serious errors would be likely to arise, if the death-rate for a single year, or for a single county or town, were taken as typical of the country as a whole. To render statistics trustworthy, they must be extended over a considerable period of time, and over a large extent of country, so as to eliminate the accidents due to a particular time or to a particular locality.

When this has been done, however, and statistics have been obtained that have a right to be regarded as really typical, the chances in any individual instance can be readily shown. Thus we find that out of one thousand children born, about two hundred and fifty die before the age of six years. The chances, then, at birth, that any child will reach this age, are $\frac{750}{1000}$ or $\frac{3}{4}$. Again, it is found that only about two persons in one thousand live to be ninety years old. So that the probability of any child living to this age would be expressed by the fraction $\frac{2}{1000}$ or $\frac{1}{500}$. This is essentially the principle upon which life insurance companies proceed. Their business is conducted on the assumption that there will be an approximately constant death-rate, though they cannot foretell what particular individuals are to die in any year. It thus becomes possible to calculate what losses from death may be expected each year. Suppose that it is found that the annual death-rate among men of a certain age throughout the country is twenty out of every thousand. If each man's life were insured for $1000, the loss to the company from this source would be $20,000. To compensate for this loss, the company would be obliged to demand an annual payment of $20 from each of the one thousand individuals in the class. Of course, the actual computations upon which insurance is based in concrete

cases are vastly more complex than this, and many other considerations arise of which account has to be taken. But the general principle involved is, that by taking a sufficiently large number of cases, chance can be almost eliminated. We can have no means of determining whether any healthy individual will or will not die before the end of the year. There would be a very serious risk, amounting practically to gambling, in insuring his life alone. But the transaction, as we have seen, is no longer a mere speculation when a large number of individuals are concerned; for the actual loss can be accurately foretold and provided for.

References

C. Sigwart, *Logic*, §§ 101, 102.
J. G. Hibben, *Inductive Logic*, Ch. XV.
L. T. Hobhouse, *The Theory of Knowledge*, Pt. II. Ch. XI.
J. S. Mill, *Logic*, Bk. III. Ch. XVIII.
B. Bosanquet, *Logic*, Vol. I., pp. 128 ff.

CHAPTER XV

METHODS OF OBSERVATION

Determination of Causal Relation

§ 53. **Mill's Experimental Methods.** — So far, we have been dealing with the methods employed in discovering the nature of particular things. We have been considering how our knowledge of the qualities and quantities of objects may be made as exact and complete as possible, but almost nothing has yet been said regarding the connection of things. Our experience, however, is not made up of isolated facts and events. We can scarcely be said to know at all, until we become aware that certain parts of our experience are united, like the links of a chain, one part involving another. And, as has been already frequently pointed out, the growth of knowledge is constantly bringing to light new connections between facts that were previously taken to be independent of each other. Of these principles of connection, the most universal and important is that of cause and effect. Thus we say that everything which happens has its cause, and is in turn followed by its effect. What rule, or rules, can now be given which will enable one to discover what is the cause or the effect of an event in any particular case?

Before we proceed to the answer of this question, however, it is necessary to explain briefly what is meant in science by the relation

of cause and effect. As the terms are used in modern scientific investigation, a cause of any phenomenon is that which necessarily and invariably precedes it; and an effect is what follows, in the same uniform way, some event which has gone before (cf. § 84). To determine the causal relation between phenomena, then, is to discover what events or circumstances always accompany each other as antecedent and consequent. Now, as will appear when we come to describe the methods actually employed, it is very often impossible to do this by means of direct observation. Reasoning and experiment have oftentimes to be summoned to the aid of observation in distinguishing between events which are merely accidentally conjoined, and those which are necessarily connected as cause and effects. But, as has been already shown (§§ 48, 49), there is no hard and fast distinction to be made between methods of observation and methods of explanation. To discover the invariable antecedent of a phenomenon is at least the beginning of explanation. Thus B is explained to some extent when I am able to point to A as its invariable antecedent. Nevertheless, since this connection of A and B is itself a fact which may be observed, its discovery may, I think, be fairly said to belong to observation rather than to explanation. Explanation, in its complete form, carries one beyond the mere fact of connection to its reasons. At the stage we have now reached, however, the problem is to show what other phenomenon, or group of phenomena, is necessarily and uniformly connected with a given event or circumstance.

The methods by which such a law of connection may be established were first formulated by Mill in his *Logic*. He stated, in general terms, the principles which were already in use in scientific procedure. Mill gives five separate canons, but, as he himself recognizes, there are but two main principles involved. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an

invariable law are two in number: One is by comparing together different instances in which the phenomenon occurs. The other is by comparing together instances in which the phenomenon does occur with instances in other respects similar in which it does not. These two methods may be respectively denominated the Method of Agreement, and the Method of Difference."[1] Of the other three methods mentioned by Mill, one — the Joint Method of Agreement and Difference — is, as the name implies, a direct combination of the first two, while the Method of Residues and the Method of Concomitant Variations are corollaries from the same principles. We shall now proceed to state and illustrate these canons.

§ 54. **The Method of Agreement.** — The principle upon which this method proceeds is stated in the following way by Mill: "*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstances in which alone all the instances agree is the cause (or effect) of the given phenomenon.*" The purpose of this rule, it will be remembered, is to help us to determine what particular facts in our experience are connected as causes and effects. If the problem is to find the cause of some phenomenon, the canon may be illustrated in the following way. Let P^1, P^2, P^3 represent different instances of a phenomenon, P, whose cause is to be ascertained. And suppose that we are able to analyze,

[1] Mill, *Logic*, Bk. III. Ch. VIII. § 1

the antecedents of P^1 into *abcd;*
the antecedents of P^2 into *gfcm;*
the antecedents of P^3 into *klnc.*

Now it is clear that *c* is the sole circumstance in which the antecedents of all these instances of P agree. We should be justified in concluding, therefore, according to this method, that *c* is probably the cause of the phenomenon under investigation, P. We may, then, adopt Jevons's formula for discovering the cause of any given phenomenon by this method: "*The sole invariable antecedent of a phenomenon is probably its cause.*"

If, now, we wished to discover the effect of something which happens, it would be necessary to determine, by observing a number of instances, what common circumstance can be found among the events which follow it.

If Q^1 were followed by *fghk,*
and Q^2 were followed by *lmgc,*
and Q^3 were followed by *grst,*

we should be able to say that Q and *g* were connected as cause and effect. The rule might then be expressed: *The sole invariable consequent of a phenomenon is probably its effect.*

When antecedents and consequents are thus represented schematically by means of letters, it is easy to perceive at once the common circumstance in a number of instances. But the facts and events of the real world are not separated off from each other in this way. The common circumstance in which a number of instances agree has to be separated out by analysis from the varia-

ble elements which form part of the different antecedents and consequents. In order to discover the common characteristic, it is necessary that we should be able to analyze a complex phenomenon into its constituent parts, and should also be able to recognize the common element as common, though it may appear in wholly different circumstances. This will become evident by considering a number of concrete cases in which this method may be employed.

If a number of cases of typhoid fever were to appear at about the same time in a community, one would naturally wish to explain this phenomenon by tracing it to its cause; and to do this one would try to discover some circumstance which was the common antecedent of all the cases. The water supply might first be examined. But if it were found that this were derived from entirely different sources in the different cases, we should probably conclude that the explanation must be sought elsewhere. Suppose that as a result of careful analysis it was discovered that all the individuals prostrated with the fever had eaten oysters bought at the same market. If this were the only common circumstance discoverable after careful investigation, we should conclude that probably the oysters were the cause of the fever. The process of analysis could be pushed still further, if one wished, in order to determine more exactly the precise source of the infection; *e.g.*, it might be found, as a result of further inquiry, that the water in which the oysters were kept was vitiated by a sewer.

Another example of the method of agreement which is often quoted by logicians may be given. One would

naturally suppose that the colours and lines of mother-of-pearl were due to the chemical or physical character of the substance itself. Sir David Brewster, however, happened to take an impression of a piece of mother-of-pearl in beeswax and resin, and was surprised to see the colours reproduced upon its surface. He then took a number of other impressions in balsam, gum-arabic, lead, etc., and found the iridescent colours repeated in every case. In this way he proved that the colours were caused by the form of the substance, and not by its chemical qualities or physical composition. The different substances, wax, balsam, lead, etc., in which the phenomenon of colour appeared, had nothing in common except the form. This, therefore, according to the method of agreement, was properly regarded as the cause of the phenomenon to be explained.

An example of the application of this method to the discovery of the *effect* of a phenomenon may now be given. Let us suppose that the problem is to determine the effect of some proposed legislation. It is necessary, of course, to refer to other instances where this legislation has been put in force. Let us suppose that in one case what followed the enactment of the law under consideration was falling off of revenue, increase of immigration, good crops, etc., and in a second, revival of ship-building, rainy weather, and increase of immigration; and that in other instances where still other conditions prevailed, the number of immigrants still continued to increase. Since this latter circumstance is the only one which follows invariably upon the enactment of the law, we are justified in concluding, after a

certain number of observations, that it is necessarily connected with the law as its result. It is important to note that the conclusions reached by this method are greatly strengthened by increasing the number of observations, and by taking instances as dissimilar in character as possible.

The method of Agreement by itself, however, is not able to afford us certainty in every case. We have spoken of the cause as 'the invariable antecedent,' and of the effect as 'the invariable consequent.' So long, then, as we are dealing with events which follow each other, there is no difficulty in perceiving which is cause, and which effect. But we are often called upon to investigate the relation between phenomena that are more permanent in character. And it is then not at all easy to determine by means of the method of Agreement which is cause and which is effect. Poverty and intemperance, for example, are found conjoined so frequently as to make it evident, apart from other considerations, that some causal relation exists between them. It might be maintained with apparently equal show of reason, that the former is the cause, or the effect, of the latter. Again, is one to say that ignorance is the cause or the effect of moral degradation? There seems to be no method of determining which is antecedent and which consequent. As a matter of fact, it is probably true in such cases that the phenomena act and react upon each other: that each term, in other words, is at once both cause and effect.

There is still another circumstance which renders uncertain the results of the method of Agreement. We have proceeded on the assumption that the given phenomenon is always produced by the same cause; and, on the other hand, that the effects of different causes are always different. But this is not so; heat, for example, may be caused by combustion, or by friction, or electricity. The fact that an effect may be produced by any one of several causes, is what is meant by the phrase 'Plurality of Causes.' Again, neither the cause nor the effect need be composed of a simple phenomenon,

or single circumstance, as has been supposed. Indeed, so far as observation can show, antecedents and consequents usually seem to consist of complex sets of circumstances. The difficulty with the method of Agreement is that it does not push the process of analysis far enough to enable us to establish completely a law of causal relation. The fact of Agreement between phenomena often serves, however, to *suggest* a law of connection. This law has afterwards to be tested by the other methods, especially by the method of Difference.

§ 55. **The Method of Difference.** — According to the method of Agreement, we compare a number of diverse instances, in all of which a given phenomenon occurs, and endeavour to discover some circumstance which is invariably present. The method of Difference, on the other hand, compares an instance in which a phenomenon occurs with another as nearly similar to it as possible, in which it does not occur. Its canon is expressed by Mill as follows: "*If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect or the cause or an indispensable part of the cause, of the phenomenon.*" It will perhaps make the matter clearer to say: 'whatever alone is present in a case when the phenomenon to be investigated occurs, and absent in another when that phenomenon does not occur, other circumstances remaining the same, is causally connected with that phenomenon.' That is, by means of this method we compare two instances which differ only in the fact that the phenomenon in which we are interested, is present in the

one, and absent in the other. If now the two cases are represented in this way,

PHK conjoined with *alg*,
and HK conjoined with *lg*,

we conclude at once that P is causally connected with *a*.

Almost any instance in which experiment is employed will serve to illustrate this method. If a bell is rung in a jar containing air, the sound will of course be heard at any ordinary distance. But after having removed the air by means of an air-pump, let the bell be again struck. It will now be found that the sound is no longer heard. When the two cases are compared, it is at once evident that the only difference in the antecedents is the presence of the air in the one case, and its absence in the other. When the air was present, the sound was heard; when it was absent, the sound was not heard. We conclude, therefore, that the perception of sound is causally connected with the presence of atmospheric air. Again, we can prove that the so-called 'taste' of different objects depends upon smell, by tasting, say, an orange, and after a little time has elapsed, tasting it a second time while holding the nose. It will be found in this latter case that instead of the familiar 'orange taste,' one senses merely 'acid,' or 'sweet.' The only difference in the two trials being that in the former the organ of smell, which was excluded in the latter, was operative, the so-called 'orange taste' is proved to be due to smell rather than to taste proper.

An essential requirement of the method of Difference

is that *only one circumstance shall be varied at a time.* The object of the method is to isolate the various conditions which go to make up a complex phenomenon, in order that we may mark the effect of the presence or absence of each one individually. Now, in observing what goes on in nature, we rarely find changes in which but a single element has varied. If we find that to-day is cooler than yesterday, we may be inclined to refer the change to the thunder-storm of last night. But rain also accompanied the thunder-storm, and the direction of the wind has changed. So that it is impossible in such cases to apply the method of difference. To employ this method successfully, observation usually must be supplemented by experiment. In performing experiments, we determine what conditions are to be operative, and arrange the apparatus so as to carry out our purpose. Having thus control of the conditions, we are able to vary them at pleasure. In this way, experiment becomes an instrument by means of which analysis can be carried further than is possible for unaided observation. It enables us to separate things which are usually conjoined, and to observe the result of each when taken by itself. In employing experiment, however, the greatest care must always be taken to introduce only one new condition at a time, or at least only one new circumstance which can in any way influence the result.

It often happens, too, as Jevons points out, that the experimenter is not aware of all the conditions which are operative when his investigations are made. "Some substance may be present, or some power may be in action which escapes the most vigilant examination.

Not being aware of its existence, we are of course unable to take proper measures to exclude it, and thus determine the share which it may have in the results of our experiments."[1] For this reason, it is always necessary that experiments should be repeated by different persons, and so far as possible under varying conditions. I quote two examples from the work of Jevons to which reference has just been made.

"One of the most extraordinary instances of an erroneous opinion due to overlooking interfering agents is that concerning the increase of rainfall near the earth's surface. More than a century ago it was observed that rain gauges placed upon church steeples, house-tops, and other elevated places, gave considerably less rain than if they were on the ground, and it has very recently been shown that the variation is most rapid in the close neighborhood of the ground. All kinds of theories have been started to explain this phenomenon; but I have attempted to show that it is simply due to the interference of wind which deflects more or less rain from all the gauges which are at all exposed to it.

"The great magnetic power of iron renders it a constant source of disturbance in all magnetic experiments. In building a magnetic observatory great care must be taken that no iron is employed in the construction, and that no masses of iron are near at hand. In some cases, magnetic observations have been seriously disturbed by the existence of masses of iron in the neighborhood. In Faraday's experiments upon feebly magnetic or diamagnetic substances, he took the greatest precautions against the presence of any disturbing substance in the copper wire, wax, paper, and other articles used in suspending the test objects. It was his invariable custom to try the effect of the magnet upon the apparatus in the absence of the object of experiment, and without this preliminary trial no confidence could be placed in the results."[2]

[1] Jevons, *Principles of Science*, Vol. II. p. 37.
[2] Jevons, *op. cit.* pp. 40, 41.

CHAPTER XVI

METHODS OF OBSERVATION

Determination of Causal Relation (continued)

§ 56. **The Joint Method of Agreement and Difference.**— When it is not possible to obtain experimental proof directly, recourse is often had to what Mill has called the joint method of Agreement and Difference. This writer has given the following expression of the canon: "*If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.*" This method, as the name implies, is a combination of the two already described. We may perhaps simplify Mill's canon somewhat by putting the matter in the following way: *A number of diverse instances having been examined, if it is found that there is a single circumstance invariably present when the phenomenon under investigation is present, and invariably absent when the latter is absent, this circumstance is causally connected with that phenomenon.* By the help of this method, the weakness which has already been noticed in the method of Agreement is overcome. We first

compare different instances in which the phenomenon occurs. If these are found to agree in only a single circumstance, we conclude, according to the canon of Agreement, that this circumstance is probably connected causally with the phenomenon in which we are interested. But the proof is not yet complete. To really prove the connection, we must show that whereever this circumstance is absent, there the phenomenon is also absent.

As an illustration of this method, we may take the case where one is trying to decide whether some stimulant like coffee or tobacco is injurious to him or not. If a person invariably found himself troubled with insomnia or nervousness after smoking, and if this seemed to him the only circumstance in his mode of life common to all these occasions, he might suspect that this was the cause. That is, the coincidence or agreement between the habit and ill-health would suggest a causal relation. But as yet, the relation would be only *suggested*, not *proved*. The method of Agreement, as we have already seen, only gives us probable conclusions. Here, however, we have the conditions under our control, and can resort to experiment and the method of Difference, in order to verify or disprove the suggestion. If after having given up smoking for a reasonable length of time, a man found that the disagreeable symptoms still continued, he would conclude that his suspicion was unfounded. But if it were found that his insomnia and nervousness had disappeared during his period of abstinence, and if the sole circumstance common to all these days and nights of exemption was the absence of smoking, he would be

forced to admit, however reluctant he might be to do so, that the troublesome physiological derangements were probably connected with the smoking habit.

§ 57. **The Method of Concomitant Variations.** — The methods of Agreement and Difference are employed, as we have seen, to determine what events are necessarily connected as causes and effects. By examining a considerable number of instances, and by comparing the cases in which the phenomenon of interest to us occurs, with cases in which it does not occur, we seek to rule out all accidental and unessential conjunctions. But as yet nothing has been said of *quantitative* relations. The discovery of a quantitative agreement or correspondence between two phenomena, or two groups of phenomena, often enables us to detect a causal relation between them (cf. pp. 192–193). Moreover, science does not rest satisfied with the mere discovery and description of changes, and the order in which they occur. We may almost say that science does not exist until the quantitative aspects of phenomena are taken into account — until things are weighed and measured. The physicist does not think his work finished when he has proved that sound is produced by atmospheric vibrations. He carries on his analysis until he can discover the *quantitative* relations between the amplitude and velocity of the vibrations, and the loudness and pitch of the resulting tone. And the psychologist is not satisfied with the general statement that certain sensations are causally connected with certain kinds of stimulus; but he seeks to discover, whenever possible, the exact quantitative relation between sensation and stimulus. In short, the

most important feature, the very essence, one may say, of modern scientific investigation, is the establishment of quantitative relations.

Looking at two things from the standpoint of quantity, then, we say that when their variations keep pace with each other, they are in some way causally connected. The following is Mill's statement of the canon: "*Whatever phenomenon varies in any manner whenever another phenomenon varies in a particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.*" The illustrations of this law given by Jevons are so excellent that we cannot do better than adopt them: —

"The illustrations of this law are infinitely numerous. Thus Mr. Joule, of Manchester, conclusively proved that friction is a cause of heat by expending exact quantities of force by rubbing one substance against another, and showed that the heat produced was exactly greater or less in proportion as the force was greater or less. We can apply the method to many cases which had previously been treated by the simple method of difference; thus instead of striking a bell in a complete vacuum, we can strike it with a very little air in the receiver of the air-pump, and we then hear a very faint sound which increases or decreases every time we increase or diminish the density of the air. This experiment conclusively satisfies any person that air is the cause of the transmission of sound.

"It is this method which often enables us to detect the material connection which exists between two bodies. For a long time it had been doubtful whether the red flames seen in total eclipses of the sun belonged to the sun or moon; but during the last eclipse of the sun, it was noticed that the flames *moved with the sun*, and were gradually covered and uncovered by the moon at successive instants of the eclipse. No one could doubt thenceforth that they belonged to the sun.

"Whenever, again, phenomena go through *Periodic Changes*, alternately increasing and decreasing, we should seek for other phenomena which go through changes in exactly the same periods, and these will probably be a connection of cause and effect. It is thus that the tides are proved to be due to the attraction of the moon and sun, because the periods of high and low, spring and neap tides, succeed each other in intervals corresponding to the *apparent* revolutions of those bodies round the earth. The fact that the moon revolves upon its own axis in *exactly* the same period that it revolves round the earth, so that for unknown ages past the same side of the moon has always been turned toward the earth, is a most perfect case of concomitant variations, conclusively proving that the earth's attraction governs the motions of the moon on its own axis.

"The most extraordinary case of variations, however, consists in the connection which has of late years been shown to exist between the Aurora Borealis, magnetic storms, and the spots on the sun. It has only in the last thirty or forty years become known that the magnetic compass is subject at intervals to very slight, but curious movements; and that, at the same time, there are usually natural currents of electricity produced in telegraph wires, so as to interfere with the transmission of messages. These disturbances are known as magnetic storms, and are often observed to occur when a fine display of the Northern or Southern Lights is taking place in some part of the earth. Observations during many years have shown that these storms come to their worst at the end of every eleven years. . . . Close observations of the sun during thirty or forty years have shown that the size and number of the dark spots, which are gigantic storms going on upon the sun's surface, increase and decrease exactly at the same periods of time as the magnetic storms upon the earth's surface. No one can doubt, then, that these strange phenomena are connected together, though the mode of the connection is quite unknown. . . . This is a most remarkable and extensive case of concomitant variations."[1]

§ 58. The Method of Residues. — We have said that

[1] Jevons, *Lessons in Logic*, pp. 249–251.

modern science employs measurement whenever possible, in order to determine exactly the quantitative relations of phenomena. Groups of facts whose connections are at first not perceived, or at best but vaguely apprehended, are brought into close relations with each other by the establishment of definite quantitative relations. The knowledge that electricity possesses energy, for example, is very vague and incomplete when compared with the definite equations which the physicist can furnish between the electrical current generated under certain definite conditions, and the amount of work which it is capable of performing. But the discovery of quantitative relations not only renders our knowledge more perfect and complete, it also enables us in some cases to detect laws of connection which would not otherwise be observed. We have already seen how the perception of corresponding changes in the quantities of phenomena has led to the discovery of causal laws by means of the method of Concomitant Variations. The method of Residues, which we now have to discuss, is also a method of quantitative determination.

In general, this method calls attention to any remainder or residue which is left over after other portions of a complex phenomenon have been explained. There are two results of this method which may be discussed separately.

(*a*) The application of this method to a complex phenomenon which is the result of several causes, often enables us to determine what part each of these causes plays in the determination of the whole fact under consideration. Mill's fifth canon seems to apply

to this case. It is as follows: *Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.* Thus, if it is known that the complex phenomenon BAC is the result of *bac*, and if it is further known that *a* is the cause of A, and *b* of B, it follows, of course, by subtraction that the residue still unexplained, C, is caused by *c*, the remaining antecedent.

Of course the application of this method in concrete cases does not usually resolve itself into such a simple process of subtraction. It requires work — 'previous inductions,' as Mill says — to determine what are the whole number of antecedents in any case, as well as to isolate the various antecedents so as to determine exactly what part of the effect is to be ascribed to each one. This may be illustrated by an example: after my student's lamp has been lighted two hours, I find the thermometer has risen from 65° to 70° Fahr. The phenomenon to be explained then is the additional 5° of heat. There is no fire, and it seems that the increase in temperature must be due to the lamp, and the heat given off from my body during this period. Suppose that the lamp is burned for the same length of time while the room is unoccupied, all other conditions remaining the same, and that the thermometer shows an increase of 4° in the temperature. By subtraction we could conclude that the heat given off by the body on the former occasion was the cause of the additional degree of temperature.

To carry the process of analysis a step further. Let us suppose that a half pint of oil, which is composed of hydrogen and carbon, has been consumed. We could determine, by measuring the heat produced by the oxidation of the exact amount of carbon contained in one half a pint of oil, what quantity of heat is due to the combustion of the carbon contained in the oil, and, by subtraction, what must be ascribed to the burning of the hydrogen.[1]

[1] This is, of course, not strictly correct. For it leaves out of account the heat generated by the *chemical combination* of the carbon and hydrogen. It may therefore serve to illustrate a case where the method of Residues breaks down.

(*b*) The second case in which this method may be applied is where there is an unexplained remainder or residue left over after the result of all the known causes has been calculated. Mill does not distinguish between such instances and the method of simple subtraction discussed above. Since, however, the cause must explain the whole of the effect, the method of residues enjoins us to continue the search for explanation. *When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.* If, for example, it were found by actual measurement that the heat produced by the lamp, and by the body of the occupant, were not sufficient to account for the change in temperature of the room, it would be necessary to seek for some further cause to account for this unexplained remainder.

This method can scarcely be said to be more than a demand for complete and precise explanation. The attempt, however, to account for unexplained residues has led to many extremely important discoveries in science. Residual phenomena are often so obscure, and appear so uninteresting and unimportant to the ordinary mind, that they are passed over without explanation. It usually requires the eye of a scientific genius to see the importance of things which appear trivial and unessential. With Darwin, facts which might appear to an ordinary observer mere unimportant exceptions, were made the object of special attention, and often served as starting-points for his investigations. Francis Darwin, speaking of his father, says: "There was one quality of mind which seemed to be of special

and extreme advantage in leading him to make discoveries. It was the power of never letting exceptions pass unnoticed. . . . A point apparently slight and unconnected with his present work is passed over by many a man almost unconsciously, with some half-considered explanation, which is really no explanation. It was just these things that he seized upon to make a start from." [1]

Among the many important discoveries which have resulted from the investigation of some obscure and seemingly unimportant fact, we may mention that of ozone. It had been observed for a long time that the passage of electric sparks through the air is accompanied by a peculiar odour. This odour was also found near electrical machines, and was known as the 'electrical smell.' No one seemed to have attached any importance to it or to have attempted to explain it in any way, until Friedrich Schönbein, a professor of chemistry at Basel, turned his attention to the subject. The result of his investigations was the discovery of ozone, the peculiar modification of oxygen, which was the cause of the odour.

Another very striking example of the application of this method is afforded by the history of the discovery of the planet Neptune. In 1781 a new planet was discovered moving outside all the other planets by Sir William Herschel. This was the planet Uranus. When its orbit came to be calculated, it was found that it did not move as it might be expected to do according to the theory of gravitation. That is, the attraction of the sun and the known planets did not account for the path it took: it moved outwards into space further than it ought to have done. It was evident that either some mistake must have been made in the observation of the astronomers, or some unknown body must be dragging it out of its course. No traces of any such planet could be perceived, and the problem remained unsolved. In 1843, a student of St. John's College, Cambridge, named Adams, undertook to work out the movements of Uranus, to discover, if possible, the position of the body which

[1] *Life and Letters of Charles Darwin*, Vol. I. p. 125.

was pulling it out of what would otherwise be its proper path, the attractions exercised by the sun and the planets in their different positions, and to show what effect they would have in determining the orbit of Uranus. Whenever the planet was deflected outwards, it was necessary to show where the body was situated which was thus influencing it. In 1845 he was able to send a paper to the astronomer royal at Greenwich, informing him in what quarter of the heavens the new planet should be observed. When the discovery was afterwards made, it was proved that his calculations were almost exactly correct. A failure on the part of the astronomer royal to coöperate by looking through his telescope for the planet gave the prior right of discovery to a Frenchman named Leverrier. The latter worked out his calculations in the same way as Adams, and obtained almost exactly the same results. He sent these results to Professor Galle of the Berlin University on the 23d September, 1846, asking him to look in the part of the heavens which he indicated. That same evening, by following out the directions, the planet was discovered in almost the exact spot predicted.[1]

The history of this discovery illustrates as well several methods and processes which we have not yet discussed, such as the formation and verification of hypotheses. It is also interesting as showing how reason is able in certain conditions to anticipate perception. The relations and forces of the heavenly bodies had been so perfectly formulated in the law of gravitation that these two investigators, working in their studies, were able to predict not only the presence but the exact position of a planet which up to that time had never been observed.

In connection with Chapters XV. and XVI., the student is advised to read Mill, *Logic*, Bk. III. Chs. VIII. and IX.

[1] Cf. Clerke, *A Popular History of Astronomy during the Nineteenth Century*, pp. 96 ff.; Buckley, *A Short History of Natural Science*, pp. 302 ff.

CHAPTER XVII

METHODS OF EXPLANATION

Incomplete Explanation. — *Analogy*

§ 59. **Explanation by Analogy.** — We have now passed from the field of observation to that of explanation. Scientific observation, aided by experiment, as we have seen, has to determine the exact nature of the facts of experience, and the order in which those facts are connected. Explanation, on the other hand, undertakes to furnish reasons why the facts are as we find them to be. But, as has already been pointed out (§§ 48, 49), no hard and fast line can be drawn between the determination of the nature and connection of facts, and their explanation. The task which our thought is called upon to perform is to transform obscurely known and isolated facts into an orderly and consistent system of knowledge. And, to accomplish this, it is necessary, in the first place, that the facts shall be thoroughly analyzed and carefully examined; and, secondly, that they shall be grouped together according to some general principle or principles which shall make clear and intelligible the relations in which they stand to each other.

To explain, then, is just to show that some fact or group of facts is related to some other fact or group with which we are acquainted. So far as the methods we have

discussed enable us to establish connections between events, they may fairly claim to be methods of explanation. Nevertheless, although the difference between these methods and those of explanation proper is one of degree rather than of essential nature, it is important to keep it in mind. The canons which were stated in the last two chapters — what Mill named the experimental methods — are rules for determining the order and succession of particular facts. The problem before us in those chapters was to determine what particular phenomena of our experience are essentially and necessarily connected as antecedents and consequents. And for this purpose active observation, aided by experiment, suffices. It is, of course, true that these observations and experiments furnish the starting-point for explanation. But they constitute a more or less distinct step in the work of systematization which is carried on by thought. The method of Difference, for instance, enables us to say that hot water will break thick glasses when poured into them, but will not injure thin ones. 'So much for the *fact*,' we say, 'but the *explanation* is still wanting.' We must try to make the fact intelligible by going outside of it, and showing that this behaviour on the part of the glasses is simply a case or illustration of what we already know of the properties of bodies when heated. Again, the method of Concomitant variations, as we have seen from Jevons's example, has led us to believe in some causal connection between electrical storms, sun-spots, and the Aurora Borealis. In this instance, knowledge has not been able to advance beyond the fact to its explanation. No satisfactory

theory has yet been established to account for the undoubted fact that these phenomena are in some way causally connected.

In discussing methods of Explanation, we deal first with Analogy. The principle of Analogy is resemblance. The phenomenon to be explained is connected with some more familiar occurrence through some perceived or imagined likeness between the two cases. In the early stages of the history of the race, everything was explained on the analogy of human actions (cf. § 84). All natural events, that is, were supposed to be produced by superhuman agents, who were, however, endowed with essentially the same qualities as man. In the thunder, the men of a primitive age heard the voice of a god. An eclipse of the sun or moon was interpreted as a divine sign or warning. When the sea became tempestuous and lashed its shores, they believed that the sea-god was angry. In every case, they interpreted these mysterious happenings of nature by referring them to causes similar in character to those which they best understood — the motives and volitions of themselves and their fellows.

The principle of analogy is employed in the same way in modern times. It is true that we no longer think that natural events are directly caused by the action of some spiritual agent more or less like ourselves. But, when we endeavour to show that the phenomena which we are interested to explain are similar in important respects to some group of facts with whose mode of operation we are familiar, we proceed by analogy. On the basis of this similarity, we argue that the

phenomena with which we are dealing probably have the same properties, or operate in the same way, or are governed by the same laws, as the better-known facts which they resemble. The formula of analogy may be stated in this way: Two things resemble each other in one or more respects, they are therefore of the same general type or character; therefore a certain proposition which is true of the one is probably true of the other. The following example of analogy has been frequently used as an illustration: —

"We may observe a very great similitude between this earth which we inhabit, and the other planets, Saturn, Jupiter, Mars, Venus, and Mercury. They all revolve round the sun, as the earth does, although at different distances and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve around their axes like the earth, and by that means must have a like succession of day and night. Some of them have moons that serve to give them lignt in the absence of the sun, as our moon does to us. They are all in their motions subject to the same law of gravitation as the earth is. From all this similitude, it is not unreasonable to think that those planets may, like our earth, be the habitation of various orders of living creatures." [1]

The word 'analogy' at the present time is somewhat loosely used for any mark of similarity or resemblance which enables us to reason from one thing to another. "The original word ἀναλογία, as employed by Aristotle, corresponds to the word Proportion in Arithmetic; it signifies an equality of ratios, ἰσότης λόγων: two compared with four is analogous to four compared with eight. There is something of the same meaning in the technical use of the word in physiology, where it is used to signify similarity of function as distinguished from similarity of structure, which is called homology; thus the tail of a whale is analogous to the tail of a fish, inasmuch

[1] Reid, *Intellectual Powers of Man*, Essay I. Chap. III.

as it is similarly used for motion, but is homologous with the hind-legs of a quadruped. A man's arms are homologous with a horse's fore legs, but they are not analogous, inasmuch as they are not used for progression."[1]

Apart from these technical uses, what is known as analogical reasoning may, perhaps, be best defined as an argument from similar instances. In analogy, we do not stop to work out a law of connection between phenomena by comparing a number of cases, or by using any of the ordinary inductive canons. But finding a striking resemblance between some circumstance — quality, arrangement, function, etc. — in the phenomena to be explained, and some phenomena with which we are already acquainted, we used the latter as a basis for conclusions about the former. Analogy is thus an argument from examples or instances, its value depending upon the real identity in some important aspect of the cases compared. When, however, our thought is able to extend to a new case, or set of cases, some general law or principle with whose operation it is already acquainted in other instances, we have passed beyond analogy to complete explanation. In the former case, we argue from the resemblance of instances; in the latter, the thread which binds the new instance with the old is the identity of a general principle.

§ 60. **Analogy as Suggestive of Explanatory Hypotheses.** — We have shown above that analogical reasoning

[1] Minto, *Logic Inductive and Deductive*, p. 367.

depends on the resemblance which exists between individual cases or instances, and that it is not guided by any general law or principle. In the next section, however, we propose to show in more detail wherein it falls short, and why, taken by itself, it can only be regarded as incomplete explanation. Here we have to notice the important part which it plays in suggesting laws and principles. Although analogy 'sticks in the particular instances,' it leads the mind on to general laws and explanatory theories. It is thus of the greatest importance as a necessary stage on the way to complete explanation.

When we are able to discover some general resemblance between a group of phenomena which we are interested to explain, and another group whose principle of operation we already understand, our thought strives to extend the known principle and to bring the new facts under it. The unknown or unexplained facts are thus brought under a known law. It is of course true that the application of the law to a new set of facts broadens our conception of its scope, and often requires us to state it in a more adequate way. Thus the analogy which Newton perceived between the heavenly bodies falling through space and the falling of the apple towards the ground, led to the formulation in exact mathematical terms of the universal law of gravitation. Our knowledge of the various functions of plants — digestion, reproduction, etc. — has been obtained by ascribing to the various organs of the plant, purposes analogous to those which are fulfilled by the parts of animal bodies. And, in turn, the study of plant physiology has thrown light

upon animal physiology, and enlarged and modified many of its theories.

An extremely interesting instance of the part which analogy plays in suggesting possible explanations, is found in the account of the discovery of the principle of Natural Selection given by Darwin in his Autobiography. In 1837 Darwin opened a note-book for the purpose of recording all facts in any way connected with the variation of species in nature and under domestication. He first investigated the variations of plants and animals which are produced under domestication, by printed enquiries, by conversation with skilful breeders, and by extensive reading. "I soon found," he says, "that selection was the keystone of man's success in making useful races of plants and animals." When useful or pleasing varieties of plants or animals occur, the gardener or breeder preserves them, and their peculiar qualities are transmitted to their offspring. And, in a number of generations, these qualities become more pronounced through accumulation. The differences between varieties of the same species of domesticated animals — varieties which are as different, for example, as the mastiff and Skye terrier — are due to the selective agency of man. But is there anything analogous takes place on an indefinitely larger scale in nature? If so, what is it which plays the part of the gardener or breeder, and preserves certain varieties?

When Darwin had reached this point in his investigations, and had come to appreciate what selection could do, he happened to read Malthus's book, *On Population*. The purpose of this book was to dispel the optimistic ideas of some of the writers of the eighteenth century who looked for the speedy realization of social well-being and happiness. Such an ideal is impossible of fulfilment, said Malthus, because of the inevitable tendency of population to increase faster than the supply of food. Human beings increase in a geometrical ratio; the means of subsistence, at best, only by an arithmetical ratio. The population will thus constantly tend to exceed the limit of the food supply, and will be kept in check only by starvation. A constant struggle for food is the lot, then, to

which each individual is doomed in virtue of this law. Darwin's observations of the rate at which plants and animals tend to reproduce their kind, led him at once to extend Malthus's principle to the whole of nature. The fecundity of natural beings leads to a struggle for existence, not merely among men, but throughout the whole organic world. And if there is a struggle, we have natural selection or the survival of the fittest. Darwin saw "that natural selection was the inevitable result of the rapid increase of all organic beings." It is not difficult to see that this discovery was the result of Darwin's wonderful power of perceiving analogies between different classes of facts. His genius led him to recognize first the resemblance of the variations of species in nature, to the more familiar variations which go on among domesticated plants and animals. And, secondly, he perceived that the competition for the means of subsistence, which the pressure of population imposes upon the members of the human race, is simply one phase of 'the struggle for existence,' which is going on everywhere throughout the organic world.

§ 61. **The Incompleteness of Analogical Reasoning.** — The most striking feature of analogical arguments is found in the fact that they yield only probable conclusions. And the reason for this is not far to seek. For, as has been already shown, analogy is a method of reasoning from one particular case to another on the basis of some imagined or perceived similarity between the two cases. Complete logical demonstration, or certainty, however, is attained only when the new fact or group of facts is really and essentially united by means of some general principle with what is already known.

But it must not be forgotten that 'probability' is not a fixed quantity. An argument from analogy may have any degree of value, from zero almost up to the limit of complete logical certainty. To *fully* explain or

demonstrate any fact, we are obliged, I think, to go beyond analogy, and to verify its conclusions by a method which has still to be described. It is evident, nevertheless, that the value of an analogical argument will depend upon the nature of the resemblance which is taken as the basis of inference. In general, it is true that the greater the resemblance between the two cases, the more certainly can we reason from one to the other. This is not to say, however, that the value of the conclusion is in direct proportion to the number of points of resemblance which can be discovered. For example, we might reason : These two men are of the same height, of the same age, live in the same house, come from the same town ; the one man stands well in his classes, therefore the other probably does so also. If the *number* of points of resemblance were the essential thing, the argument ought to possess some weight, but it is clear that it has none. The difficulty is that none of the resemblances mentioned are fundamental, or in any way essential to the real nature of the things compared. If we knew that the two men were similar in character, this one characteristic would be worth more, as a basis for the conclusion, than all the circumstances which we have mentioned combined.

It is true, then, as Mr. Bosanquet remarks, that in analogical reasoning we must *weigh* the points of resemblance rather than count them.[1] Other things being equal, the more points of resemblance we can make out the better; but if these are to contribute at

[1] *Logic*, Vol. II., p. 99.

all to the certainty of the conclusion, they must represent some deep-lying characteristic of the things compared. In general, it must be said that it is only experience which can inform us what resemblances are fundamental, and what merely external. Systematic knowledge in any field enables us to separate the essential from the accidental. And, what is perhaps a corollary from this, it must not be forgotten that the value of an inference from analogy depends largely upon the amount of intellectual insight possessed by the mind which makes it. The ordinary mind, at least in its undisciplined and untutored condition, regards all things as of equal importance. It is therefore led away by the strongest stimulus — by striking external and accidental resemblances. On the other hand, a scientific genius whose mind is well stored with facts, and who is gifted in addition with imagination, is able to penetrate beneath the surface and to apprehend the real or fundamental resemblance. His imagination enables him to see beyond the chaos of the particular facts, and to detect the underlying principle by means of which these facts can be connected and systematized.

Analogy thus becomes deepened until it passes from the stage of a mere argument from particular to particular, to the perception of a general law which includes the individual instance. But no such direct insight can claim the title of knowledge, until it is tried and tested by the facts. The guesses of scientific men unfortunately often prove mistaken. It is always necessary that fancy shall be confronted with facts. Even Dar-

win's magnificent analogical inference was nothing more than a hypothesis, as he himself well understood, until its power of explaining the facts of organic life was demonstrated. We have now to explain in the next chapter the methods by which such guesses are tested.

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CHAPTER XVIII

METHODS OF EXPLANATION.—THE USE OF HYPOTHESES

§ 62. **Reasoning from an Hypothesis.** — An hypothesis is a guess or supposition made to explain some fact or group of facts. We have seen in the last chapter how the mind is led on by the perception of analogies to formulate a general law or principle of explanation for phenomena which were not previously understood. But even when guided by analogy, a guess or hypothesis is only the beginning of explanation. A mere hypothesis or supposition must be tried by its capacity to explain facts, and in this way either verified or disproved. 'Theory' is another word that is often used as equivalent to hypothesis. Strictly speaking, however, it is more correct to use the term 'hypothesis' for the unverified, or only partially verified guess, and to reserve 'theory' for the hypothesis that has been more completely demonstrated. This distinction, however, is not usually maintained, and even in scientific writings the terms 'theory' and 'hypothesis' are used interchangeably. Nevertheless, it is necessary to distinguish in some way the 'mere hypothesis,' or supposition, which is quite as likely to be false as true, from the hypothesis which has been established by proof.

It is well to remember that it is not only in solving scientific problems that we employ hypotheses. In our

ordinary experience, we are constantly trying to imagine the most likely explanation of facts which we perceive through the senses. If, for example, one should find on returning to one's room that a pane of glass had been broken, one would straightway set about finding some explanation of this occurrence. One might perhaps first imagine that a stone or something of the kind had been thrown against it. Acting on this supposition, one would look for the stone in the room. If it were found there, the hypothesis would be confirmed; if no traces of it could be discovered, and if, moreover, on examination the glass proved to be shattered in a way that would probably not result from the projection of a stone against it, our first hypothesis would have to be abandoned. We should then make another guess — perhaps that the outside blind had been violently closed by the wind — and again examine the facts to see if they gave any support to this supposition. We are constantly making hypotheses of this character to explain phenomena which we meet with in everyday experience. If we find a stream swollen, we conclude that it must have rained in some part of the country drained by the stream. If a man has typhoid fever, we are pretty sure to guess that he has been drinking impure water. We no sooner perceive something unusual or striking than we begin to guess out, as it were, its explanation. The formation of hypotheses, then, is simply the mind's response to the demand for explanation.

It is worth noticing that it is only unusual or striking events, or those in which they have some practical concern, which attract the attention of the majority of mankind, and lead them to form explana-

tory hypotheses. What is familiar, or of no practical importance, does not usually awaken curiosity. Indeed, in a great many cases, such phenomena are not observed at all. But the great scientist is distinguished, one may say, by his intellectual curiosity. He tries to understand phenomena which the ordinary mind neglects, and simply takes for granted. He has questions in his mind with regard to familiar things which he wishes to have answered, guesses which he is desirous of having proved or disproved. We have found it convenient, in the preceding chapters, to separate the description of the processes of determining the nature of facts, from the account of the methods of explanation. But it must by no means be supposed that the nature of facts is discovered quite independently of the influence of hypotheses or theories. Unless the mind has some question to answer, or theory to test, it is impossible to see any significance in an experiment. In other words, every experiment must have a purpose, and the purpose is to get some information that will help us to answer a question which we bring with us to the investigation.

In the actual process of acquiring knowledge, then, observation and theorizing go hand in hand. Unless we go to nature with something in our mind, we are not likely to learn much. As a rule, we see only what we look for. Francis Darwin says of his father: "He often said that no one could be a good observer unless he were an active theorizer. This brings me back to what I said about his instinct for arresting exceptions: It were as though he were charged with theorizing power ready to flow into any channel on the slightest disturbance, so that no fact, however small, could avoid releasing a stream of theory, and thus the fact became magnified into importance. In this way it naturally happened that many untenable theories occurred to him, but fortunately his richness of imagination was equalled

by his power of judging and condemning the thoughts which occurred to him. He was just to his theories and did not condemn them unheard; and so it happened that he was willing to test what would seem to most people not at all worth testing. These rather wild trials he called 'fool's experiments,' and enjoyed exceedingly. As an example, I may mention, that finding the cotyledons of Biophytum to be highly sensitive to vibrations of the table, he fancied that they might perceive the vibrations of sound, and therefore made me play my bassoon close to a plant." [1]

A good example of how essential theories are for an observer, and how blind he may be to what he is not looking for, is found in the work from which we have just quoted. In the brief autobiography contained in the first volume, Darwin tells of a geological trip through Wales which he took while a student at Cambridge, in company with Sedgwick, the professor of geology. It must be remembered that this was before Agassiz had come forward with his theory of a glacial period in the world's history. Darwin writes: "We spent many hours in Cwm Idwal, examining all the rocks with supreme care, as Sedgwick was anxious to find fossils in them; but neither of us saw a trace of the wonderful glacial phenomena all around us; we did not notice the plainly scored rocks, the perched boulders, the lateral and terminal moraines. Yet these phenomena are so conspicuous that, as I declared in a paper published many years afterward in the *Philosophical Magazine*, a

[1] *Life and Letters of Charles Darwin*, Vol. I., p. 126.

house burnt down by fire did not tell its story more plainly than did this valley. If it had been filled by a glacier, the phenomena would have been less distinct than they now are." [1]

§ 63. **Formation of Hypotheses.** — We are now ready to consider a little more closely the formation of hypotheses or theories. In the first place, it is to be noticed that hypotheses are not received from without through sense-perception, but are made by the mind. They are the creations of the imagination. A good theorizer, like a poet, is in a certain sense born, not made. The man to whom 'nothing ever occurs,' whose intellectual processes are never lit up with a spark of imagination, is unlikely to make any important discoveries. It has been by a flash of scientific genius, by imaginative insight which we may almost call inspiration, that great scientific theories have been discovered. Not even a scientific genius, however, can afford to neglect the facts. But, guided by accurate observation, the scientific imagination tries to invent some law or principle which will serve to connect and explain facts. Tyndall has an essay on "The Scientific Use of the Imagination," from which we may quote a short passage. "With accurate experiment and observation to work upon, imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the prepared imagination. . . . Out of the facts of chemistry the constructive

[1] *Life and Letters of Charles Darwin*, Vol. I., p. 49.

imagination of Dalton formed the atomic theory. Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, preceding, accompanying, and guiding all his experiments. His strength and fertility as a discoverer are to be referred in great part to the stimulus of the imagination. Scientific men fight shy of the word because of its ultra-scientific connotations; but the fact is, that without the exercise of this power, our knowledge of nature would be a mere tabulation of coexistences and sequences."[1]

In speaking of hypotheses as 'guesses,' or 'creations of the imagination,' their dependence upon facts must not be forgotten. It is only when the phenomena to be explained have been carefully observed that our guesses at their explanation are likely to be of value. It is well known that a considerable amount of knowledge is usually required to ask an intelligent question. And in the same way, the mind must be well stored with facts, in order to render our hypothetical explanations worthy of consideration. Indeed, observation of facts, and the formation of theories go hand in hand, and naturally assist each other. We have already spoken of the lack of theory which makes us blind to facts which seem to lie directly before us. But we have perhaps not yet emphasized sufficiently the dependence of theories upon the facts of observation. The process of explanation may be described as a fitting together of the facts given by observation, with the explanatory theories which the mind originates. The theory with which we start enables us to ask questions, and leads us to scrutinize the phenomena which are to be explained; while the latter react upon the theory, and cause it to undergo constant modification. The account of Darwin's discovery of the principle of 'the survival of the fittest' is a good illustration of an hypothesis constructed by a constant dependence upon the facts during every step of its progress.

[1] *Fragments of Science*, p. 104.

We have already referred to the way in which analogy leads the mind on to general principles of explanation (§ 60). Analogy is a method of inferring that what is true of one object is probably true of others which resemble it. But the ordinary mind sees resemblances only when they are very obvious and striking. The man of scientific insight, on the other hand, like the poet, penetrates more deeply into the nature of things, and is able to discover analogies and resemblances to which the ordinary man is blind. Who but a genius like Newton would have thought of connecting the fall of an apple with the fall of the heavenly bodies through space? The history of science shows that great discoveries are made by means of imaginative insight, but it also teaches that mere imagination without dependence upon known facts is frequently a source of much mischief. Mere theories without facts are not only empty, but often stand in the way of true knowledge. The fruitful exercise of the imagination, if we may judge from the way in which great discoveries have been made, always takes place in closest connection with what observation and experiment reveal regarding the nature of phenomena. If the imagination is to have power to discover any truth, it must constantly 'touch earth,' and be guided in its course by the nature of facts which are already known.

In framing hypotheses, then, the imagination is constantly prompted by analogies with processes which are more or less familiar. The hypothesis, then, is not created by the imagination 'out of nothing.' It is rather an extension or development of a known law, than an absolute creation.

§ 64. **The Proof of an Hypothesis.**—We have discussed the way in which hypotheses are formed, but as yet have said nothing regarding the means of determining their truth and falsity. But to form hypotheses is usually easy, to verify them is often exceedingly difficult. The scientific worker constantly finds that theories which he has formed are without foundation, and must therefore be discarded. It is not only essential that a scientific investigator shall possess a mind fertile in ideas ; he must also love truth more than any theory, no matter how interesting or attractive it may appear. In behalf of truth, every theory must be subjected to the most thorough and searching tests possible ; if it is not borne out by the facts, it must be at once discarded. What now is the general method of procedure in testing an hypothesis? Two steps or stages may be distinguished in this process : (1) We assume that the hypothesis is true, and proceed to show what are the necessary results which follow from it. In doing so we proceed deductively ; that is, assuming the truth of the hypothesis, we reason out what consequences it must have. (2) The conclusions thus reached are compared with the actual facts, as given to us directly in perception, or as determined by experiment. If these are found to agree, the hypothesis is regarded as true ; if they do not agree, it must be discarded or modified.

This procedure may become clearer by considering some concrete examples. If we were to come on the campus some morning and find that several branches had been broken from one of the trees, we should naturally try to explain this circumstance by making

some hypothesis. Perhaps the first thing which would occur to us would be that there had been a violent wind-storm. The hypothesis having been made, the next step would be to look around to see if it could be verified. 'If there has been a cyclone,' we might argue, 'there should be other signs of its presence; we should find broken twigs and blown leaves lying about, and all the trees should present a storm-tossed appearance.' If observation showed that these things were actually present, we would consider our hypothesis so far confirmed. But if not, our first guess would be disproved, and it would be necessary to look about for another explanation.

An excellent illustration of the way in which an hypothesis becomes more and more completely demonstrated, is found in the history of the experiments by which it was proved that the atmosphere has weight. Galileo noticed that water will rise in a pump only about 33 feet. He could not find out, however, why it was that the water should stop at this point. After his death, his friend and pupil Torricelli took up the problem, and asked himself : Why does the water rise at all ? It then occurred to him that air must weigh something, and that it might be this weight on the surface of the water which forced the water up the pump when there was no air pressing it down. Now, if this were so, he reasoned, the weight of the air ought to lift mercury, which is fourteen times heavier than water, to one-fourteenth of the height. So he took some mercury, and filling a tube about 34 inches long, turned it upside down into a basin of mercury which was open, and therefore under the pressure of the atmosphere. The mercury began to settle in the tube, and finally rested at a height of 30 inches. Torricelli had thus invented the barometer, an instrument which would measure the weight of the atmosphere. It was afterwards suggested by the famous French writer, Pascal, that at the top of a high mountain, where there is less

air pressing downwards, the column of mercury should fall considerably if the atmosphere were really what caused the water and the mercury to rise. When this experiment was made by carrying the barometer to the top of a mountain called the Puy de Dôme, the mercury fell nearly three inches. Still further confirmation of Torricelli's theory was afforded by the discoveries of Otto Guericke of Magdeburg. In 1650 Guericke invented the air-pump. The first use which he made of his new invention was to show that the atmosphere is pressing down upon us heavily and equally in all directions. He fitted closely together two metal hemispheres and exhausted the air between them by means of his pump. It was found that the pressure of the atmosphere was so great that it took a great force to separate the hemispheres.[1]

To establish a scientific theory, then, there are necessary not only a ready imagination, but also patience and perseverance in the careful deduction of the consequences of the theory, and in the comparison of the results thus obtained with the actual facts. Scientific work also demands the utmost candor and openness of mind on the part of those who engage in it. One must be willing to abandon any theory as soon as it is found to disagree with the facts. And this is by no means an easy thing to do. When one has a theory which suffices for nearly all the facts, there is always a temptation to cling to it, and to neglect or explain away any troublesome or contradictory facts. There is no doubt that the scientific explanations which have become accepted and established were not the ideas which first happened to occur to the men with whose names they are associated. When Newton first attempted to work out the verification of the gravitation hypothesis, he used the

[1] Cf. Buckley, *Short History of Natural Science*, pp. 114-121.

most accurate measurements he could obtain regarding the size of the earth. But in calculating on this basis the pull of the earth on the moon, and the consequent deflection of the moon from the straight line, his results came out wrong. That is, the moon moved more slowly than it ought to do according to his theory. The difference was not great, but Newton could not overlook this lack of agreement with the observed facts. He put the whole matter aside; and it was only when he heard sixteen years later that Picart had discovered, from new and more accurate measurements, that the earth was larger than had been supposed, that he repeated his calculations, and found his hypothesis verified.

Although it very frequently turns out, both in everyday matters and in scientific work, that our hypotheses are disproved, the negative answers thus obtained are not without value. For we are often able at once to limit the number of possible hypotheses. In a field where we already possess some systematic knowledge, it is often possible to say: The explanation of this group of phenomena must be either *a* or *b* or *c*. If, then, one is able to show that neither *a* nor *b* will afford the required explanation, these negative conclusions will lead directly to the establishment of *c*.

§ 65. **Requirements of a Good Hypothesis.** — Various conditions or requisites of a good hypothesis are laid down by writers on logic. The three laws which are most frequently stated are as follows: (1) That the hypothesis shall be conceivable and not absurd. (2) That it shall be of such a character that deductions

can be made from it. (3) That it shall not contradict any of the known laws of nature.

It does not seem to me that the first law is of much value. It is largely individual taste or education which leads us to pronounce certain theories 'absurd' or 'inconceivable.' Thus, for a long time, it seemed inconceivable that the earth should be round, and should revolve on its own axis; and less than a generation ago the theory of evolution, as propounded by Darwin, seemed to many persons utterly 'absurd.' Nor can the third law always be applied as a test of an hypothesis, for many great discoveries seemed, at the time when they were announced, to contradict known laws of nature. The difficulty is that no one is able to affirm, unconditionally, that a law of nature forbids us to make this or that hypothesis. Of course, we feel that a theory is very probably false which is at variance with the law of gravity, or with that of the conservation of energy, or any of the laws which we regard as established beyond a reasonable doubt. But, although the chances are always very greatly against any theory which runs counter to what are regarded as well-established laws, there is yet always a *possibility* that it may be true. There is no law of nature so certain as to be infallible. Even those laws which appear to be beyond the possibility of doubt, may require to be modified or supplemented. We may find that, practically, it is not wise to trouble ourselves with theories which undertake to overthrow the law of gravitation, or to disprove other fundamental laws of the physical world. But theoretically, at least, there is always a chance — in cases

such as we have been supposing the chance is almost infinitely small — that the new theory may be right, and the old one wrong. The practical objection to admitting the claims of this canon is the difficulty in applying it fairly. The phrase, 'contrary to the laws of nature,' like 'inconceivable,' and 'absurd,' is likely to be used to condemn any theory with which one disagrees. In this way, it is evident that the very point is begged which is really at issue.

Of these three canons, therefore, the second appears to state the only condition which is essential to an hypothesis. An hypothesis, if it is to be of any value, must be capable of being proved or refuted. But, unless its consequences can be shown by way of deduction, it is impossible to know whether it agrees, or does not agree, with the facts which it is supposed to explain. An hypothesis from which nothing can be deduced, then, is of no value whatever. It always remains at the stage of mere possibility, and without any real connection with fact. It is a mere guess which has no significance whatever, for it is entirely incapable either of proof or of disproof.

In general, it is possible to deduce the consequences of a theory only when the principle employed is analogous, in mode of operation, to something with which we are familiar. Thus, for example, it is because the ether is conceived as resembling other material bodies in important respects that it can be used as a principle of explanation. It is assumed to be elastic and capable of receiving and transmitting vibrations, and as spread out like other material bodies in space. In virtue of these similarities to other material substances, it is possible to deduce the consequences which such a substance as ether would imply, and to compare them with the

actual facts. But if one should make the assumption that certain phenomena are due to some agency totally unlike anything of which we have any experience, a disembodied spirit, or ghost, for example, it would be impossible either to prove or to disprove the assertion. For knowing nothing whatever of the way in which spirits act, one could not say whether the phenomena to be explained, table-rapping, planchette-writing, etc., were or were not consistent with a spirit's nature and habits.

Another example of a barren hypothesis from which no conclusions can be drawn, is afforded by the 'catastrophe' or 'convulsion' theory in geology, which was first combatted by Lyell, in his *Principles of Geology*, published in 1830. "People had so long held the belief that our earth had only existed a few thousand years, that when geologists began to find a great number of strange plants and animals buried in the earth's crust, immense thicknesses of rock laid down by water, and whole mountain masses which must have been poured out by volcanoes, they could not believe that this had been done gradually, and only in parts of the world at a time, as the Nile and the Ganges are now carrying down earth to the sea, and Vesuvius, Etna, and Hecla are pouring out lava a few feet thick every year. They still imagined that in past ages there must have been mighty convulsions from time to time, vast floods swallowing up plants and animals several times since the world was made, violent earthquakes and outbursts from volcanoes shaking the whole of Europe, forcing up mountains, and breaking open valleys. It seemed to them that in those times when the face of the earth was carved out into mountains and valleys, table-lands and deserts, and when the rocks were broken, tilted up, and bent, things must have been very different from what they are now. And so they made imaginary pictures of how nature had worked, instead of reasoning from what they could see happening around them."[1]

The convulsions, or catastrophes, which were thus assumed to take place were regarded as the result of strange incalculable forces whose mode of operation could never be exactly determined.

[1] Buckley, *Short History of Natural Science*, pp. 441–442.

Instead of these mysterious agencies, Lyell assumed that causes similar to those with which we are now acquainted had been acting uniformly for long ages. The nature of the causes at work being known, it became possible to calculate the nature of the effects, and thus to reduce the facts of geology to order and system. As we have already shown, hypotheses which are to prove really serviceable are formed by extending some known principle through analogy to a new class of facts. The assumption of mysterious agencies and principles whose mode of operation is unlike anything which is known to us, does not aid in the extension of knowledge.

References

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" " " *The Principles of Science*, Ch. XXIII.
C. Sigwart, *Logic*, § 83.
B. Bosanquet, *Logic*, Vol. II., pp. 155–167.

CHAPTER XIX

FALLACIES OF INDUCTION

§ 66. **The Source of Fallacy.** — It is necessary at the close of our discussion of the inductive methods, to say something regarding the errors to which we are most subject in this kind of thinking. We have seen that knowledge is the result of the mind's own activity, and that it grows in completeness through a persistent effort to keep distinct things which are different, and to connect phenomena which belong together. Truth, in other words, is gained by intellectual activity. And, on the other hand, we fall into error, and are led away by false arguments as a result of mental indolence. Thinking is hard work, and there is always a tendency to avoid it. As a matter of fact, we all think much less frequently than we suppose. Usually, we are content to follow familiar associations, and to repeat current phrases, without doing any real intellectual work. The difficulty is that we can get along comfortably without thinking for the most part — more comfortably, perhaps, than when we do think. Then, again, the mind is less directly under control of the will than the body. One may force himself to sit down at his desk and open a book; but it is more difficult to compel oneself to think.

The only way in which we can be saved from becoming 'intellectual dead-beats,' is by the formation of good

mental habits. It requires eternal vigilance and unceasing strenuousness to prevent our degeneration into mere associative machines. What the logical doctrine of fallacies can do is to put us on our guard against this tendency. It enumerates and calls attention to some of the commonest and most dangerous results of slovenly thinking, in the hope that the student may learn to avoid these errors. Some of the fallacies of which we shall treat in this chapter, apply equally to deductive or syllogistic reasoning, and have been already treated in Chapter XI. We shall, however, enumerate them here again for the sake of completeness. It is convenient to discuss the various fallacies under the following heads: —

(1) Fallacies due to the careless use of Language.
(2) Errors of Observation.
(3) Mistakes in Reasoning.
(4) Fallacies due to Individual Prepossessions.

§ 67. **Fallacies due to the Careless Use of Language.** — The careless and unreflective use of words is a very frequent source of error. Words are the signs or symbols of ideas; but the natural sluggishness of the mind leads often to a substitution of the word for the idea. "Men imagine that their reason governs words, whilst, in fact, words react upon the understanding; and this has rendered philosophy and the sciences sophistical and inactive."[1] It is much easier to deal with counters than

[1] Bacon, *Novum Organum*, Aph. LIX.

with realities. Since we must use words to express our thoughts, it is almost impossible to prevent them from becoming our masters. The dangers from the use of words has been well represented by Locke, from whom I quote the following passage: —

"Men having been accustomed from their cradles to learn words which are easily got and retained, before they knew or had framed the complex ideas to which they were annexed, or which were to be found in the things they were thought to stand for, they usually continue to do so all their lives; and, without taking the pains necessary to settle in their minds determined ideas, they use their words for such unsteady and confused notions as they have, contenting themselves with the same words other people use, as if their very sound necessarily carried with it constantly the same meaning. . . . This inconsistency in men's words when they come to reason concerning either their tenets or their interest, manifestly fills their discourse with abundance of empty, unintelligible noise and jargon, especially in moral matters, where the words, for the most part, standing for arbitrary and numerous collections of ideas not regularly and permanently united in nature, their bare sounds are often only thought on, or at least very obscure and uncertain notions annexed to them. Men take the words they find in use among their neighbours; and, that they may not seem ignorant what they stand for, use them confidently, without much troubling their heads about a certain fixed meaning; whereby, besides the ease of it, they obtain this advantage: That, as in such discourses they seldom are in the right, so they are as seldom to be convinced that they are in the wrong; it being all one to go about to draw men out of their mistakes who have no settled notions, as to dispossess a vagrant of his habitation who has no settled abode."[1]

(1) In treating of the misuse of words, we mention, in the first place, errors arising from the use of a word

[1] *Essay Concerning Human Understanding*, Bk. III. Ch. X.

or phrase in more than one sense. This is usually called the fallacy of Equivocation. In some cases, the equivocation may be mere wilful quibbling on the part of the person propounding the argument, as in the following example of Jevons: —

> All criminal actions ought to be punished by law,
> Prosecutions for theft are criminal actions,
> Therefore prosecutions for theft ought to be punished by law.

Examples of this kind do not mislead any one; but in some instances the change of meaning in words may not be perceived, even by the person who employs the argument. For example, one might reason: —

> It is right to do good to others,
> To assist A in obtaining office is to do him good,
> Therefore it is right to assist him in this way.

Here the phrase which is used equivocally is, 'to do good,' as will at once be perceived.

(2) Another frequent source of error in the use of words is found in what has been excellently named the Question-begging Epithet. As is well known, there is much in a name. Epithets like 'class-legislation,' 'compromise measure,' 'a dangerous and immoral doctrine,' are terms freely used to describe the measures or views of opponents. And, as it is always easier to adopt a current phrase, than to examine the facts and draw our own conclusions, it is not surprising that the name settles the whole matter in the minds of so many people. Of course, the epithet employed may beg the question in favour of the subject it is used to describe, as well as against it. Politicians well understand the

importance of adopting an impressive and sonorous election cry to represent the plank of their party. Thus, party cries like 'honest money,' 'prohibition and prosperity,' 'the people's cause,' etc., are essentially question-begging epithets. Even words like 'liberty,' 'justice,' and 'patriotism,' are frequently used in such a way as to bring them under the class of fallacies which we have here described. Under this heading, also, may be grouped 'cant' words and phrases. When we accuse a person of using cant, we always imply that he is more or less consciously insincere, that he is professing opinions and sentiments which he does not really possess. Any insincere expression which is made primarily for the sake of effect may be rightly termed cant. It is not even necessary that the speaker should be fully conscious of his insincerity. A man may easily deceive himself, and, as he repeats familiar words and phrases, imagine himself to be overflowing with patriotism, or with sympathy for others, or with religious feelings.

(3) Figurative language is another frequent source of error. Of the various figures of speech, perhaps metaphors are the most misleading. The imagery aroused by metaphorical language is usually so strong as to make us forget the difference between the real subject under consideration, and the matter which has been used to illustrate it. Thus in discussing problems of mind, it is very common to employ metaphors drawn from the physical sciences. For example, we read in works on psychology and ethics of 'the struggle of ideas,' of 'the balancing and equilibration of motives,' of 'action in

the *direction* of the strongest motive,' etc. Another illustration, which has been often quoted, is Carlyle's argument against representative government founded on the analogy between the ruler of a state and the captain of a ship. The captain, he says, could never bring the ship to port if it were necessary for him to call the crew together, and get a vote every time he wished to change the course. The real differences between the relation of a captain to his crew, and the executive officers in a state to the citizens, is lost sight of by the metaphor. Metaphorical reasoning is simply a case of analogy, the imperfections and dangers of which have been already pointed out. It is, however, one of the errors which it is most difficult to avoid. A hidden metaphor lurks unsuspected in many of the words in common use. We may thus appreciate the force of Heine's humorous petition: "May Heaven deliver us from the Evil One, and from metaphors." [1]

§ 68. **Errors of Observation.** — Sometimes insufficient observation is the result of a previously conceived theory; sometimes it may be due to inattention, to the difficulties of the case, or to lack of the proper instruments and aids to observation. We have already had occasion to refer to the influence of a theory on observation (cf. § 62). As a rule, we see only those instances which are favourable to the theory or belief which we already possess. It requires a special effort of attention to take account of negative instances, and to discover the

[1] Quoted by Minto, *Logic*, p. 373.

falsity involved in some long-standing belief. Indeed, it perhaps requires quite as much mental alertness to overthrow an old theory, as to establish a new one. This tendency of the mind to seize upon affirmative instances, and to neglect the evidence afforded by negative cases, is well set forth by Bacon in the following passage: —

"The human understanding, when any proposition has been once laid down (either from general admission and belief, or from the pleasure it affords), forces everything else to add fresh support and confirmation; and although most cogent and abundant instances may exist to the contrary, yet either does not observe or despises them, or gets rid of and rejects them by some distinction, with violent and injurious prejudice, rather than sacrifice the authority of its first conclusions. It was well answered by him who was shown in a temple the votive tablets suspended by such as had escaped the peril of shipwreck, and was pressed as to whether he would then recognize the power of the gods; 'But where are the portraits of those who have perished in spite of their vows?' All superstition is much the same, whether it be that of astrology, dreams, omens, retributive judgment, or the like, in all of which the deluded observers observe events which are fulfilled, but neglect and pass over their failure, though it be much more common. But this evil insinuates itself still more craftily in philosophy and the sciences, in which a settled maxim vitiates and governs every other circumstance, though the latter be much more worthy of confidence. Besides, even in the absence of that eagerness and want of thought (which we have mentioned), it is the peculiar and perpetual error of the human understanding to be more moved and excited by affirmatives than negatives, whereas it ought duly and regularly to be impartial; nay, in establishing any true axiom the negative instance is the most powerful."[1]

The nature of this fallacy has been so well illustrated

[1] *Novum Organum*, Bk. I. Aph. XLVI.

by the quotation which has just been given, that we may pass on at once to speak of other cases of insufficient observation. Our discussion of the processes of reasoning have made it clear how necessary it is to observe carefully and attentively. The majority of the false theories which have appeared in science and in philosophy, as well as those of common life, have arisen from lack of observation. The doctrine of innate ideas, and the theory that combustion was a process of giving off phlogiston — a substance supposed to be contained in certain bodies — may be given as examples. In some seaside communities, there is a belief that living beings, both human and animal, never die at flood tide. 'They always go out with the ebb,' it is said. Again, there is a general belief, which was shared by such an eminent scientist as Herschel, that the full moon in rising possesses some power of dispersing the clouds. Careful observations made at the Greenwich observatory have, however, shown conclusively that the moon has no such power as that supposed.[1]

Another circumstance to be considered in this connection is the inaccuracy and fallibility of ordinary memory. Every one must have noticed how rarely two persons agree completely in the report which they give of a conversation which they have heard, or of events which they have experienced. This is due in part to diversity of interest: each person remembers those circumstances in which for any reason he is most strongly interested. But, in addition, it is largely the result of

[1] Cf. Jevons, *Principles of Science*, Ch. XVIII.

the inevitable tendency of the mind to confuse what is actually observed, with inferences made from its observations. The inability to distinguish between what is really perceived, and what is inferred, is most strongly marked in uneducated persons, who are not on their guard against this fallacy. An uneducated person is certain to relate, not what he actually saw or heard, but the impression which the events experienced made upon him. He therefore mixes up the facts perceived, with his own conclusions drawn from them, and with statements of his own feelings in the circumstances. A lawyer who has to cross-examine a witness is usually well aware of this tendency, and takes advantage of it to discredit the testimony. The experienced physician knows how worthless is the description of symptoms given by the ordinary patient, or by sympathetic friends, or by an inexperienced nurse. The more one's sympathies and interests are aroused in such a case, the more difficult it is to limit oneself to an exact statement of actual occurrences.

But this tendency is not confined to persons deficient in knowledge and ordinary culture. It usually requires special training to make one a good observer in any particular field. It is by no means so easy as it may appear to describe exactly what one has seen in an experiment. If we know, or think that we know, the explanation of the fact, there is an almost inevitable tendency to substitute this interpretation for the account of what has been actually observed. Recent psychological investigation, aided by exact experimental methods, has done much to disentangle the data of

perception from inferences regarding these data. As every one knows who has practised psychological introspection, it is only with the utmost difficulty, and after long training, that one can distinguish the actual psychological process present to consciousness, from the associative and logical elements which are bound up with them in our ordinary experience. The following passage from Mill deals with this question: —

"The universality of the confusion between perceptions and the inferences drawn from them, and the rarity of the power to discriminate the one from the other, ceases to surprise us when we consider that in the far greater number of instances the actual perceptions of our senses are of no importance or interest to us except as marks from which we infer something beyond them. It is not the colour and superficial extension perceived by the eye that are important to us, but the object of which these visible appearances testify the presence; and where the sensation itself is indifferent, as it generally is, we have no motive to attend particularly to it, but acquire a habit of passing it over without distinct consciousness, and going on at once to the inference. So that to know what the sensation actually was is a study in itself, to which painters, for example, have to train themselves by long-continued study and application. In things further removed from the dominion of the outward senses, no one who has not had great experience in psychological analysis is competent to break this intense association; and when such analytic habits do not exist in the requisite degree, it is hardly possible to mention any of the habitual judgments of mankind, from the being of God and the immortality of the soul down to the multiplication table, which are not, or have not been, considered as matter of direct intuition."[1]

§ 69. **Mistakes in Reasoning.** — The problem of the inductive processes of reasoning is to ascertain what

[1] *Logic*, Bk. V. Ch. IV. § 5.

facts are necessarily and essentially connected, and to explain this connection. Now, in order to distinguish between chance conjunctions of phenomena, and real causal connections, careful and extensive observation, aided whenever possible by experiment, must be employed. In short, to establish a real law of connection between phenomena, it is necessary to use one or more of the inductive methods described in Chapters XIV. and XV. But to do this implies, in many cases, long processes of analysis; the performance of intellectual work, which ordinary minds, at least, have the tendency to shirk whenever possible. It is much easier to allow associations to control our thoughts, and to assume that events which happen together in our experience a number of times are causally connected. We are led to such a conclusion by a natural psychological tendency, without taking any thought about the matter, while logical analysis and discrimination require a distinct conscious effort.

The general name used to describe fallacies which are due to this particular form of mental sluggishness is *post hoc, ergo propter hoc.* Two events occur in close conjunction with each other, and it is then assumed without further investigation that they are related to each other as cause and effect. Many popular superstitions, are examples of this fallacy. Some project begun on Friday turns out disastrously, and it is inferred that some causal relation existed between the fate of the enterprise, and the day on which it was begun. Or thirteen persons sit down to dinner together, and some one dies before the year is out. It is to be noticed that

such beliefs are supported by the tendency, to which we referred in the last section, to observe only the instances in which the supposed effect follows, and to neglect the negative cases, or cases of failure. 'Fortune favours fools,' we exclaim when we hear of any piece of good luck happening to any one not noted for his wisdom. But we fail to take account of the more usual fate of the weak-minded. The belief that the full moon in rising disperses the clouds, which was also quoted earlier, is a good example of *post hoc, propter hoc*. In fact, all the fallacies treated in this chapter, except those due to language, might quite properly be included under this heading.

A special case of this fallacy, to which attention may be called separately, arises from hasty generalization, or generalization on an insufficient basis of fact. There is a constant tendency on the part of the mind to seek general conclusions, to express all its knowledge in the form of general statements. But, although it is the aim of science to express the truth regarding the nature of the world in the form of general laws, it is not allowable to hurry on to such principles without first making our observation of the facts as complete as possible. Thus it is not unusual to hear a traveller declare, on the basis of a very limited experience, that 'the hotels of some city or country are thoroughly bad.' The generalizations which are so frequently made regarding the peculiar characteristics of Americans, or Englishmen, or Frenchmen are usually of the same sort. Conclusions regarding the effect of moral and political conditions, too, are often drawn from observations in

a limited field. Even scientific books are not always free from this error. In a recently published psychological study of the first year of the life of a child, by the mother, it was explained why a baby always sucks its thumb rather than its fingers. The explanation was that the thumb, being on the outside and projecting outwards, got oftenest into the baby's mouth, and so the habit was formed. The point is, that the mother assumed what she had observed in her own child to be true universally. Other parents, however, declare that their babies never put the thumb into the mouth, but always the fingers or the whole hand.

§ 70. **Fallacies due to Individual Prepossessions.** — Bacon named this class of fallacy "The Idols of the Cave." Each individual, as he represents the matter, is shut up in his own cave or den; that is, he judges of things from his own individual point of view. In the first place, one's inclinations and passions, likes and dislikes, pervert one's judgment. It is exceedingly difficult, as we all know, to be fair to a person we dislike, or to refrain from judging too leniently the shortcomings of those to whom we are warmly attached. Again, it is not easy to put oneself in the position of an impartial spectator when one's interests are at stake. "The understanding of men," says Bacon, "resembles not a dry light, but admits some tincture of the passions and will." Furthermore, each individual has a certain personal bias as a result of his natural disposition and previous training. Thus it is almost impossible for an individual to free

himself from national prejudices, or from the standpoint of the political party, or the church in which he was brought up. Or, if a person does give up his old views, he not infrequently is carried to the opposite extreme, and can see no good in what he formerly believed. Even education and the pursuit of special lines of investigation may beget prejudices in favour of particular subjects. When a man has been engaged exclusively for a long time in a particular field, employing a particular set of conceptions, it is almost inevitable that he should look at everything with which he has to do in the same light. The mathematician's view of the world is almost sure to be different from that of the historian, or that of the student of æsthetics. It is very difficult for the physicist to conceive of any natural process except in terms of molecules and vibrations. It is inevitable that each man should be blinded to some extent by his own presuppositions. But to recognize one's limitations in this respect, is to pass, to some extent at least, beyond them.

Moreover, each age, as well as each individual, may be regarded as governed largely by current presuppositions and prejudices. Throughout the Middle Ages, theological doctrines and opinions controlled almost absolutely the opinions and beliefs of mankind. This influence, doubtless, still makes itself felt, but people are now pretty generally awake to the dangers from this source. On the other hand, it is more difficult to realize at the present time that it is not impossible for prejudices and prepossessions to grow out of scientific work. The success of modern scientific methods has sometimes led investigators to despise and belittle the work of those who do not carry on their investigations in laboratories, or do not weigh and measure everything. But conceptions and methods

which prove useful in one science cannot always be employed profitably in another. A conception, or mode of regarding things, which has proved serviceable in one field is almost certain to dominate a whole age, and to be used as an almost universal principle of explanation. The eighteenth century, for example, was greatly under the influence of mechanical ideas. Newton's discovery made it possible to regard the world as a great machine, the parts of which were all fitted together according to the laws of mechanics. This view led to such a vast extension of knowledge in the realm of physics and astronomy, that the conceptions upon which it is based were applied in every possible field — to psychology, to ethics, to political science. The world itself, as well as religious creeds and political and social institutions, were supposed to have been deliberately made and fashioned by some agent. Again, in these later years of the nineteenth century we are dominated by the idea of evolution. The biological notion of an organism which grows or develops has been applied in every possible field. We speak, for example, of the world as an organism rather than as a machine, of the state and of society as organic. And the same conception has been found useful in explaining the nature of human intelligence. It is easy for us to realize the limitations and insufficiency of the notion of mechanism as employed by the thinkers of the eighteenth century. But it is not improbable that the twentieth century may be able to see more clearly than we are able to do, the weaknesses and limitations of the conception which has proved so fruitful in this generation.

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PART III.—THE NATURE OF THOUGHT

CHAPTER XX

JUDGMENT AS THE ELEMENTARY PROCESS OF THOUGHT

§ 71. **Thinking the Process by which Knowledge grows or develops.** — Logic was defined (§ 1) as the science of thinking, and we have seen that the business of thought is to furnish the mind with truth or knowledge. Under what general conception, now, shall we bring thinking, and what method shall we adopt to aid us in its investigation? It is at once clear that thinking, the conscious process by which knowledge is built up, does not resemble mechanical processes like pressure, or attraction and repulsion. It is more nearly related to something which has life, like a plant or an animal, and which grows or develops from within, in accordance with the laws of its own nature. Thinking must be regarded rather as a living, than as a dead thing, though it is necessary also to remember that it is conscious as well as living.

When the thinking process is regarded in this way, moreover, a method of procedure at once suggests itself. In these days we have become familiar with the notion of evolution or development, and the application of this

notion has proved of the greatest service to science, and particularly to those sciences which deal with the phenomena of life. What is characteristic of this manner of regarding things is the fact that it does not consider the various phenomena with which it deals as fixed, unchangeable things, each with a ready-made nature of its own. But each thing is simply a stage of a process, a step on the way to something else. And the relations of the various phenomena to each other, their connection and unity as parts of the one process, come out more clearly when viewed in this way. In other words, by taking a survey of the genesis and growth of things, we gain a truer idea of their nature and relations than would be possible in any other way. The past history of any phenomenon, the story of how it came to be what it is, is of the greatest possible service in throwing light upon its real nature. Now, one cannot doubt that this conception will also prove serviceable in the study of logic. That is to say, it will assist us in gaining a clearer idea of the nature of thinking, to conceive it as a conscious function, or mode of acting, which unfolds or develops in accordance with the general laws of organic evolution. And this process may be supposed to go on both in the individual, as his thought develops and his knowledge expands, and in the race, as shown by its history. By adopting this notion, we may hope to show also that there is no fundamental difference in kind between the various intellectual operations. Judgment and Inference, for example, will appear as stages in the one intellectual process, and the relation between Induction and Deduction will become evident.

§ 72. **The Law of Evolution and its Application to Logic.** — The most striking characteristic of any organism at a low stage of development is its almost complete lack of structure. An amœba, for example, can scarcely be said to have any structure; it is composed of protoplasm which is almost homogeneous, or of the same character throughout. When we compare an amœba, however, with an animal much higher in the scale of life, *e.g.*, a vertebrate, a great difference is at once evident. Instead of the simple, homogeneous protoplasm, the organism is composed of parts which are unlike or heterogeneous, such as bones, muscles, tendons, nerves, blood-vessels, etc. In Mr. Spencer's language, there has been a change from a state of homogeneity, to one of heterogeneity. The process of evolution from the lower organism to the higher has brought with it a differentiation of structure. That is, in the amœba there are no special organs of sight, or hearing, or digestion, but all of these acts seem to be performed by any part of the organism indifferently. In the vertebrate, on the other hand, there is division of labour, and a separate organ for each of these functions. One may also notice that the same change is observable when the *acts or functions*, performed by a lower organism are compared with those of a higher. The life of the amœba seems to be limited almost entirely to assimilation and reproduction; while, when we advance from the lower animals to the higher, and from the higher animals to man, there is an ever-increasing complexity and diversity in the character of the actions performed. We thus see how the process

of evolution involves differentiation both of *structure* and of *function*, in passing from the homogeneous to the heterogeneous.

But differentiation, or increase in diversity, is only one side of the process of evolution. As we pass from a lower to a higher stage, the various parts of an organism are seen to become more essential to each other. If certain plants or low animal organisms are divided into several parts, each part will go on living. Its connection with the other parts does not seem to have been at all necessary to it. But when we are dealing with higher forms of life, each part is seen to have its own particular function, and to be essential to the other parts, and to the organism as a whole. In other words, the parts now become members, and the whole is not simply an aggregation of parts or pieces, but is constituted by the necessary relation of the members to each other. The more highly evolved the whole with which we are dealing, the more closely connected and essential to each other are the various parts seen to be. It becomes increasingly true that if one member suffers, all the other members suffer along with it.

Evolution, then, not only exhibits a constant process of differentiation, and a constant increase in the diversity of parts and organs, but there goes along with this what might be called a process of unification, whereby the parts are brought into ever closer and more essential relation to one another. In this way, a *real or organic whole*, as opposed to a mere *aggregate*, is formed. This is what Mr. Spencer calls the process of **integration**; and it accompanies, as we have seen, what the same writer calls **differentiation**.

The application of this general law of evolution to the development of the thinking process is not difficult. We shall expect to find that thinking, in its first beginnings, both in the individual and in the race, will be much less complex than at a higher stage. That is, the earliest or simplest thinking tends to take things in a lump, without making any distinctions. The infant, for example, does not distinguish one person from another, or perhaps does not distinguish even the parts of its own body from surrounding objects. Now, it is clear that intellectual development, growth in knowledge, must in the first place involve differentiation. What is complex must be analyzed or separated into its various parts. Things which are different must be distinguished, and clearly marked off from each other. The development of thought implies then, as one of its moments, discrimination or analysis — what we previously called differentiation.

The other moment of the law of evolution, integration, also finds a place in the development of thought, and goes hand in hand with the former. The child and the uneducated man not only often fail to make distinctions where these really exist, but the parts of their knowledge are fragmentary, and have little or no relation to one another. The various pieces of their knowledge are like the parts of the amœba — they may be increased or diminished without themselves undergoing any change. But in order to pass from a lower to a higher intellectual point of view, — to become better educated, in a word, — it is necessary to see the way in

which the various pieces of our knowledge are connected and depend upon one another. It is not enough to analyze and keep separate things which are distinct, but it is also necessary to understand how the various parts of our knowledge are so related as to be essential to one another. In other words, we may say that it is characteristic of our intelligence to endeavour to put things together so as to form a whole, or system of interconnected parts. And the more completely it is able to do this (provided that the process of differentiation has also made a corresponding advance), the higher is the stage of development which has been attained. The ideal of knowledge, or of complete intellectual development, would be to understand the oneness and relation of everything which exists, even of all those things which seem now to be entirely different in kind. A knowledge of any one fact would then carry with it a knowledge of every other fact. Or, rather, our knowledge would be so completely unified, that each part would show the nature of the whole or system to which it belongs; just as a leaf of a plant, or the tooth of an animal, is sufficient to tell the naturalist of the wholes to which they belong.

This, of course, will always remain an ideal; but it is in this direction that thinking actually develops. It is a step in advance to discover the reasons for any fact which one previously knew as a mere fact. But, to discover the reasons for a fact, is to bring it into connection with other facts, to see them no longer as isolated and independent, but as belonging together to one group or system of facts. And the further

the process of explanation goes on, the more completely is our knowledge unified and related.

There is, however, another fact implied in the very nature of evolution, of which logic, as well as the other sciences, may take advantage. We have assumed that the more complete and difficult kinds of thinking have grown or developed from simpler types of the same process, and not from something different in kind. It will therefore follow, that the essential characteristics of the thinking process may be discovered in its simplest and most elementary form. It is found that all the essential functions of the fully developed organism are discharged by the primitive cell. And because it is easier to study what is simple than what is complex, the cell is taken as the starting-point in biology. Similarly, there will be an advantage in beginning with the simplest and most elementary forms of thinking. What is found true of these simple types of thought, may be assumed to be essential to the thinking process as such.

§ 73. **Judgment as the Starting-point.** — What, then, is the simplest form of thinking? What shall we take as a starting-point, which will correspond to the cell in biology, or the elementary process in psychology? To answer this question, it is not necessary first to decide where in the scale of animal life that which we are entitled to call thinking actually begins. We shall not be obliged to discuss the much-debated question, whether or not dogs think. Wherever thinking may be found, it is essentially an activity of the mind. When it is present, that is, there is always work done, something

interpreted or put together, and a conclusion reached. One may perhaps say that thinking is simply the way in which the mind puts two and two together and sees what the result is. It implies that the mind has waked up to the significance of things, and has interpreted them for itself. Suppose that one were sitting in one's room very much engaged with some study, or wrapt up in an interesting book, and suppose that at the same time the sound of a drum fell upon one's ears. Now, the sound-sensations might be present to consciousness without calling forth any reaction on the part of the mind. That is, we might be so intent on our book that we should not wake up, as we have been saying, to the meaning or significance of the drum-taps; or perhaps not even to the fact that they were drum-taps at all. But if the mind did react upon the sound-sensations, it would try to interpret them, or put them together so as to give them a meaning. As a result, some conclusion would be reached, as, for example, 'the drum is beating'; or sufficient intellectual work may have been done to give as a conclusion, 'that is the Salvation Army marching up the street.' In any case, it is of the greatest importance to notice that the conclusion does not come into our minds from without, but that it is the product of the mind's own activity, as has been described. It is not true, in other words, that knowledge passes into our minds through the senses; it is only when the mind wakes up to the meaning of sensations, and is able to put them together and interpret them, that it gains any knowledge.

Now, the simplest form of such an act of thought is

called a judgment. Judgment, we may say, is a single intellectual act of the kind we have described; and its conclusion is expressed by means of a Proposition; as, for example, 'the grass is green,' 'the band is playing.' In accordance with general usage, however, we may use the term 'Judgment' for both the act itself and its result. And the word 'Proposition' will then denote the external expression in speech or writing of the product of an act of judgment.

In our investigation of the nature of thought, then, we must begin with Judgment. There are three things which we shall have to do: (1) to endeavour to discover the fundamental characteristics of this simple type of thinking; (2) to show the various forms which it assumes, or to describe the different kinds of Judgment; and (3) to trace the process by which Judgment expands into the more complete logical form of Inference. Before any of these questions are considered, however, it is necessary to meet a very serious objection to our whole procedure of beginning with Judgment as the elementary process of thinking.

§ 74. **Concepts and Judgments.** — In the last section, we endeavoured to show that Judgment is the elementary process of thought, and that with it all knowledge begins. This view, however, may seem to be contradicted by the treatment of Judgment usually found in logical text-books. Judgment, it is said, is expressed by a proposition; and a proposition is made up of three parts, subject, predicate, and copula. Thus in the proposition, 'iron is a metal,' 'iron' is the subject, 'a metal'

the predicate, and the two terms are joined or united by means of the copula 'is.' A Judgment is therefore defined as an act of joining together, or, in negative judgments, of separating, two concepts or ideas. If this account be accepted, it follows that the ideas of which the judgment is composed (iron and metal, in the example given above) are pieces of knowledge which precede the judgment itself. And the act by which these logical ideas (or, as they are usually called, concepts) are formed must also be earlier and more fundamental than the act of judging. It is therefore held that logic should begin with concepts, which are the elements out of which judgments are compounded, and that the first logical act consists in the conception or simple apprehension of the ideas or concepts (cf. § 11).

It is necessary to examine this position very carefully. What is maintained is that a process of forming concepts, or logical ideas, presumably quite distinct from the activity of judgment, necessarily precedes the latter. Before it is possible to judge that 'iron is a metal,' for instance, one must have gained, by means of Conception or Apprehension, the ideas denoted by the ~~subject and predicate of this~~ proposition. Judgments, that is, are made or compounded out of something different from themselves.

It may be well to begin the defence of our own position by noting what is undoubtedly true in what has just been stated. In making a judgment like 'iron is a metal,' it is, of course, necessary to have the concept 'iron,' and the concept 'metal.' But what is implied in having a concept of anything? Let us

suppose that a person is making the above-mentioned judgment for the first time — that is, really drawing a conclusion for himself, and not merely repeating words. He would begin, we may say, with the concept 'iron.' But if this concept is more than a mere word, if it really means anything, it must have been formed by a number of judgments. The concept 'iron,' if it has any significance for the person using it, means a definite way of judging about some substance — that it is hard, malleable, tough, etc. The greater the number of judgments which the concept represents, the more meaning or significance it has; apart from the judgment, it is a mere word, and not a thought at all.

To admit, then, that in judging we always start from some concept, does not imply that there is a different form of intellectual activity prior to judgment, which furnishes the latter with ready-made material for its use. But, as we have seen, in ordinary judgments like the example with which we have been dealing, the new judgment is a further expansion or development of a previous set of judgments which are represented by the concept. The concept, then, stands for the series of judgments which have already been made. Language comes to the aid of thought, and makes it possible to gather up such a set of judgments and represent them by a single expression — often by a single word. Every word that is the name of some logical concept represents intellectual work — the activity of judgment — in its formation. In learning our own language, we inherit the word without doing the work. But it must never be forgotten that the word in itself is not the

concept. To make the thought our own, to gain the real concept, it is necessary to draw out or realize to ourselves the actual set of judgments for which the word is but the shorthand expression.

The view which regards the judgment as a compound of two parts — subject and predicate — rests upon the substitution of words for thoughts. It analyzes the *proposition* (the verbal or written expression of the judgment), instead of the judgment itself. In the proposition, the parts do exist independently of each other. The subject usually stands first, and is followed by the predicate. But there is no such order of parts in a judgment. When one judges, 'it is raining,' or, 'that is a drum,' the piece of knowledge is one and indivisible. And the act by which this knowledge is gained, is not an external process of joining one part to another, but is an intellectual reaction by which we recognize that something, not previously understood, has a certain meaning or significance.

Again, it is only when concepts are identified with the words which make up the parts of the proposition, that they can be regarded as ready-made existences, which are quite independent of their connection in a judgment. The terms, 'iron,' and 'metal,' are separable parts of the proposition and exist independently of their connection with it. The conclusion has been therefore drawn that concepts had a like independence of judgments, but might enter into the latter and form a part of them without affecting their own nature in any way. But, as we have already seen, the concept has no meaning apart from the series of judgments which it

represents. And, as thinking goes on, as new judgments are made, its nature is constantly changing. In short, concepts are not dead *things*, but living *thoughts* which are in constant process of development.

The objection, then, which urges that conception is a logical process, which is prior to judgment, turns out when rightly understood to be no objection at all. For, in the light of what has been already said, it only amounts to this: In making new judgments regarding anything, we must set out from what we already know of it, as represented by the judgments already made. That is, the starting-point for a new judgment is the concept or series of judgments which represents the present state of our knowledge. The progress of knowledge is not from the unknown to the known, but from a state of partial and incomplete knowledge to one of greater perfection. Thus the judgment 'gold is malleable' (supposing it to be a real judgment made for the first time), adds to, or develops further, our existing knowledge of gold, as represented by a series of judgments previously made regarding it.

It may be urged, however, that not every judgment can grow out of previous judgments in this way. For, if we go back far enough, we must reach some judgment which is absolutely first, and which presupposes no antecedent judgment. This is like the paradox regarding the origin of life. If all judgments are derived from antecedent judgments, how was it possible for the first one to arise? It will, perhaps, be sufficient answer to deny the existence of the paradox. Consciousness must be regarded as having from the first the form of a judgment. No matter how far one goes back in the history of consciousness, one will always find, so long as consciousness is present at all, some reaction, however feeble, upon the

content, and something like knowledge resulting. Even the consciousness of the newly born infant, reacts, or vaguely judges, in this way. These primitive judgments are, of course, very weak and confused, but they serve as starting-points in the process of intellectual development. Growth in knowledge is simply the process by means of which these vague and inarticulate judgments are developed and transformed into a completer and more coherent experience.

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CHAPTER XXI

THE MAIN CHARACTERISTICS OF JUDGMENT

§ 75. **The Universality of Judgments.** — We have now to examine the nature of Judgment a little more closely than we have done hitherto. And, in the first place, we note that all judgments claim universality. There are, however, several kinds of universality, and more than one sense in which a judgment may be said to be universal. We speak of a universal judgment (more properly of a universal proposition), when the subject is a general term, or is qualified by some such word as 'all,' or 'the whole.' And we distinguish from it the particular judgment, where the subject is only the part of some whole, and is usually preceded by 'some,' or by other partitive words. But here we have no such distinction in mind; we are speaking of the universality which belongs to the very nature of Judgment as such, and which is shared in by judgments of every kind.

When we say that judgments are universal in the sense in which the word is now used, we mean that the conclusions which they reach claim to be true for every one. No matter what the subject and the predicate may be, a judgment, *e.g.*, 'man is mortal,' comes forward as a fact for all minds. We have shown in the last chapter that it is by judging, or putting things together for itself, that the human mind gains knowledge. Now,

the assumption upon which this process is based is that the result thus reached—knowledge—is not something merely individual and momentary in character. When I judge that 'two and two are four,' or that 'iron has magnetic properties,' the judgment is not merely a statement of what is going on in my individual consciousness; but it claims to express something which is true for other persons as well as for me. It professes to deal with facts which are true, and in a sense independent of any individual mind. The judgments by which such conclusions are reached are universal, then, in the sense of being true for every one and at all times. The word 'objective' has essentially the same meaning. Although each man reaches truth only by actually judging for himself, yet truth is objective, out there beyond his individual or 'subjective' thought, shared in by all rational beings. The assumption upon which all argument proceeds is that there is such a standard, and that if people can be made to think they will arrive at it. Thought is objective, or, in other words, has in itself its own standard of truth.

The only alternative to this position is scepticism, or pure individualism. If Judgment is not universal in the sense that it reaches propositions which are true for everybody, it is of course impossible to find any standard of truth at all. The judgments of any individual in that case would simply have reference to what seems true to him at the moment, but could not be taken to represent any fixed, or permanent truth. Indeed, if one regards Judgment as dealing merely with particular processes in an individual mind, the ordinary meanings of truth and falsehood are completely lost, and it becomes necessary to give a new definition of the words. This was the position of the Sophists at the time of Socrates (cf. § 5). Each

individual man was declared to be the measure of what is true and false, as well as of what is good and bad. There is thus no other standard of truth or value than the momentary judgment (or caprice) of the individual. This is, in a way, the *reductio ad absurdum* of scepticism.

The common nature of truth, as something in which all can share, presupposes, then, a common mode of thinking or judging on the part of all rational beings. And it is this universal type or form of knowing with which logic deals. The question as to whose thought is investigated, or in what individual mind the thought takes place, is in itself of no importance. The consciousness of a savage differs very greatly from that of an educated man; it is much less complex and less highly developed. But yet, in spite of the enormous differences, there exists in both an intelligence, or way of thinking, which shows the same essential character, and operates according to the same fundamental laws.

§ 76. **The Necessity of Judgment.** — The second characteristic which we note as belonging to Judgment is **necessity**. By this we mean that when a person judges, he is not free to reach this or that conclusion at will. As an intellectual being, he feels bound to judge in a certain way. This is sometimes expressed by saying that we cannot believe what we choose, we must believe what we can.

In many of the ordinary judgments of everyday life, which are made without any clear consciousness of their grounds, logical necessity is implicitly present as an immediate feeling of certainty. In cases of this kind, we simply identify ourselves with the judgment, and *feel* that it is impossible that it can be false. But, of course, no judgment can claim to be necessary in its own right. Its necessity comes from its connection with other facts

which are known to be true. Or, in logical terms, we may say that it comes from reasons or premises which support it. And one should always be ready to show the grounds or reasons upon which one's feeling of necessity rests. But in ordinary life, as we have seen, it is not unusual to regard a conclusion as necessary, without clearly realizing the nature of the reasons by which it is supported. An uneducated man is rarely able to go back and discover the reasons for his belief in any statement of which he is convinced. If you question his assertion, he feels that you are reflecting upon his veracity, and consequently grows angry. In the feeling of immediate necessity or conviction, he identifies himself with the judgment, and does not see that the criticism is not directed against the latter, but against the grounds by which it is supported.

In this distinction between necessity that is merely *felt*, and the necessity that is *conscious* of its own grounds, we see the direction in which judgment must develop. In the evolution of thought, we must become conscious of the grounds upon which our judgments are made. That is, the simple judgment, which seems to stand in isolation, must expand so as to unite with itself its reasons. By itself, it is only a fragment of a more complete and widely embracing thought. The feeling of necessity is an evidence of its dependence and connection, though this dependence and connection upon other facts may not be clearly understood. But what is implicit must be made explicit; the necessity which is merely *felt* to belong to the simple judgment must

be justified, by showing the grounds or reasons upon which it rests. And, for this purpose, the simple judgment must expand so as to include the reasons which are necessary to support it. In other words, it must develop into an inference. As a matter of fact, the same form of words as used by different persons, or by the same person at different times, may express either a judgment or an inference. Thus, 'the price of wheat rose after the war began,' might express either a simple historical fact, which is accepted from experience or from hearsay, or it might, in the mouth of a person acquainted with the laws of supply and demand, be the necessary conclusion of a number of premises. Again, a child might read that, 'the travellers found great difficulty in breathing when they reached the top of the mountain,' accepting this as a simple statement of fact. If he were to read this same statement some years later, however, he would probably connect it at once with other facts regarding the nature of the atmosphere, and the action of gravity, and so perceive at once its inferential necessity.

According to the view which has just been stated, necessity is not a property which belongs to any judgment in itself, but something which arises through its dependence upon other judgments. In other words, necessity is always mediate, not immediate. This view, however, differs from a theory that was once generally received, and has some adherents, even at the present time, especially among thinkers who belong to the Scottish or 'common-sense' school. In dealing with the facts of experience, we always explain one fact by referring it to a second, and that second by showing its dependence upon some third fact, and so on. Thus the movement of the piston-rod in an engine is explained by the pressure of steam, and this is due to the expansive power of heat, and heat is caused by combus-

tion of fuel, etc. We are thus pushed back in our explanations from one fact or principle to another, without ever reaching anything that does not require in its turn to be explained.

Now, it is said that this process cannot go on forever; for if it did there could be no final or complete knowledge; the whole system would be left hanging in the air. There must, therefore, it is argued, be some ultimate facts which furnish the support for the world of our experience, some principle or principles which are themselves necessary and do not require any proof. That is, there must be certain propositions which are *immediately* necessary, and which serve as final explanation for everything else. Now, it is clear that such propositions must be entirely different in character from the ordinary facts of experience, since their necessity belongs to their own nature, and is not derived from any other source. It had to be supposed, therefore, that they stood upon a different plane, and were not derived from experience. To explain the superior kind of certainty which they were assumed to possess, it was supposed that they were present in the mind at birth, or were innate. They have also been called *necessary truths*, *a priori truths*, and *fundamental first principles*, in order to emphasize their supposed distinction from facts which are derived from experience.

§ 77. **Judgment involves both Analysis and Synthesis.**— The business of our thought is to understand the ways in which the various parts of the real world are related. And a judgment, as we have already seen, is just a single act of thought,—one step in the process of understanding the world. Now we ask: How does Judgment accomplish its task? Does it proceed by analysis, showing the parts of which things are composed, or does it employ synthesis in order to show how various parts combine in such a way as to form a whole? Or is it possible for both these processes to be united in one and the same act of judgment?

Suppose that one actually makes the judgment for oneself (and does not merely repeat the words), 'the rose has pinnate leaves.' What has taken place? We notice, firstly, that a new property of the rose has been brought to light; a distinction, or mark, has been discovered in the content 'rose,' which was not seen to belong to it before the judgment was made. So far, then, the process is one of analysis, of discovering the parts or distinctions of something which is at first taken, as it were, in a lump. And this is a most essential element in all thinking. In order to know, it is absolutely necessary that the differences between the parts of things should be clearly apprehended, that we should not confuse things which are unlike, or fail to make proper distinctions. If we examine a number of instances where a real judgment is made, we shall find that this moment of analysis, or discrimination, is always present. Sometimes, indeed, analysis may not seem to be the main purpose of the judgment; but if one looks closely, one will always find in a judgment that elements which are unlike are held apart or discriminated.

Let us look again at the same judgment, 'the rose has pinnate leaves.' It is not difficult to see that the discovery of something new in itself is only one part of what the judgment has accomplished. The judgment also affirms the union of this new discovery with the properties of what we call the rose. It is, therefore, from this point of view, an act of synthesis. It asserts that the prickly branches, fragrant flowers, feather-like leaves, and other distinctions, are united in the one content which we call the rose. It does not stop with

the mere assertion, 'there is a mark or distinction,' but it affirms that it is a mark of something, *i.e.*, that it is united with other marks or properties to form a concrete whole. In other words, we may say that every judgment affirms the *unity* of the different parts, or aspects, of a thing; and this is, of course, synthesis. From this point of view, then, Judgment can be defined as a process of synthesis, just as we defined it above as one of analysis.

But how, it may be asked, is it possible for a judgment to be both analytic and synthetic? Are not these processes directly opposed to each other? There can be no doubt that this is the case when we are dealing with material things: pulling things to pieces is the opposite of putting them together. When we are doing the one we cannot also be doing the other. But there is no such opposition between these processes when they go on in our minds. An illustration may make this clear. Suppose that one is trying to understand some piece of mechanism, say a watch; in order to be able to see how it goes, or judge correctly regarding it, two things are necessary. First, one must notice all the parts of which it is composed — the wheels of various sizes, springs, pins, etc. But, in the second place, one would not understand the watch until one saw how all the parts were united, how one part fits into another, and all combine together into one whole. We do not mean that these are two steps which take place in succession; as a matter of fact, the detection of the various parts, and the perception of their connection, go hand in hand. In the process of understanding

the watch, we have both taken it to pieces and put it together again at one and the same time. Not really, of course, but in our thought. In the world of material things, as we have said, only one of these processes could go on at a time; but in every act of thinking, in every judgment, analysis and synthesis go hand in hand, and one has no meaning except with reference to the other.

Although every judgment contains, as we have seen, the two moments of analysis and synthesis, these are not always equally prominent. The main purpose of the judgment usually falls on one side or the other. In a judgment like, 'water can be divided into hydrogen and oxygen,' the main emphasis seems to be on the parts, and the assertion that these elements are *parts of a whole*, though present, is only implied. But when one asserts, 'these springs and wheels together make up a watch,' it is the nature of the whole upon which the emphasis is laid, and the separation or discrimination of the parts, is, as it were, secondary. It is not difficult to see, however, that the two moments of Judgment are present in both of these cases. The difference consists in the fact that at one time analysis, and at the other synthesis, is made the main purpose.

It was at one time supposed that analytic and synthetic judgments were entirely different in kind from each other. An analytic judgment, it was said, is one in which the predicate is obtained by analyzing, or bringing to light, what is contained in the subject. Thus the judgment, 'all material bodies fill space,' is analytic: for the predicate (space-filling) is contained in

the very notion, or idea, of a material body. All that is necessary in order to obtain the judgment is to comprehend the meaning of the subject. An analytic judgment, then, adds nothing to our knowledge. It merely enables us to bring to light and express what is contained in the ideas we already possess. A synthetic proposition, on the contrary, was defined as one in which the predicate was not already contained in the subject, but which added a new element or idea to it. 'This body weighs ten pounds,' for example, is a synthetic proposition, for one cannot obtain the predicate by analyzing the subject. The predicate adds a new fact which must have been derived from experience.

This view is of course fundamentally different from the account of Judgment which we have just given. The absolute distinction between analytic and synthetic judgments, like the theory that thought begins with concepts, arises, I think, from a substitution of the spoken or written proposition for the judgment itself. In the proposition the subject seems to be the starting-point. We have a word or term which appears to be independent and capable of standing alone. The question is, then, where shall we find the predicate? For example, in the proposition, 'iron is an element,' the subject stands first, and the predicate comes later. It seems possible then to say that we have first the subject 'iron,' and then join on to it the predicate 'element,' which has been obtained either by analyzing the subject, or from some previous experience. But the proposition, as a collection of words, must not be substituted for the act of judgment. Judgment, as we have already seen, is a single act of intelligence, which at once discriminates and brings into relation different aspects of the whole with which it is dealing. A mere subject by itself has not any intelligible meaning. If one hears the word 'iron,' for example, the word may call up certain mental images; but by itself it is not a complete thought or fact in

which we can rest. 'Well, what of it?' we say. The mind at once goes on to form some judgment like, 'this is iron,' or 'iron is heavy.' We cannot *think* a term without thinking something *of* it. In short, although the words which form the subject of a *proposition* are relatively independent, and can be used without the words which make up the predicate, in a *judgment*, on the other hand, a subject is only a subject *through* its relation to a predicate. The proposition may be divided into parts, but the judgment is a single thought-activity, and cannot be divided (cf. § 74).

§ 78. **Judgment as Constructing a System of Knowledge.** In this section we have not to take account of any new characteristic of Judgment, but rather to emphasize the part it plays in building up knowledge. As we have seen, Judgment works both analytically and synthetically: it discovers new parts and distinctions, and at the same time brings the parts into relation and thus builds up a whole. That is the law according to which thinking develops, and is just what we called differentiation and integration in a previous section (§ 72).

It is necessary here, however, to dwell upon the fact that each judgment may be regarded as a step in the process of building up a *system* of knowledge. The emphatic word here is 'system,' and we must be perfectly clear about its meaning. A system is a whole which is composed of various parts. But it is not the same thing as an aggregate or heap. In an aggregate or heap, no essential relation exists between the units of which it is composed. In a heap of grain, or pile of stones, one may take away any part without the other parts being at all affected thereby. But in a system, each part has a fixed and necessary relation to the whole

and to all the other parts. For this reason we may say that a building, or a piece of mechanism, is a system. Each stone in the building, each wheel in the watch, plays a part, and is essential to the whole. In things which are the result of growth, the essential relations in which the parts stand is even more clearly evident. The various parts of a plant or an animal have each their own function, but at the same time they are so necessary to each other that an injury to one is an injury to all. We express this relation in the case of living things by saying that the parts are *organic* to each other. And, in the same way, it is not unusual to speak of society as an organism, in order to express the fact that the various individuals of which it is composed are not independent units, but stand in necessary relations to one another, and are all mutually helpful or hurtful.

We have said that Judgment constructs a *system* of knowledge. This implies, then, that it is not merely a process of adding one fact to another, as we might add one stone to another to form a heap. No! Judgment combines the new facts with which it deals, with what is already known, in such a way as to give to each its own proper place. Different facts are not only brought together, but they are arranged, related, systematized. No fact is allowed to stand by itself, but has to take its place as a member of a larger system of facts, and receive its value from this connection. Of course, a single judgment is not sufficient to bring a large number of facts into relation in this way. But each judgment contributes *something* to this end, and brings some new fact into relation to what is already known.

In a simple judgment like, 'that was the twelve o'clock whistle,' the constructive or systematizing work accomplished is evident. The auditory sensation, which in itself, as a mere wandering sound, was not a piece of knowledge at all, is interpreted in such a way as to find a place in the system of experience. One may appreciate what part the judgment really plays by remembering how the sound appeared before one was able to judge. There may have been at first a moment of bewilderment—'What does this mean?' one asks. In the next moment the judgment is made: 'It is the twelve o'clock whistle.' That is, our thinking has constructed a meaning for it, and brought it into relation with the rest of our knowledge.

(1) Every new experience is thus brought into relation with the facts which we already know, and is tested by them. It has to find its place in the system of knowledge—to join itself to what is already known. If this is impossible, if what claims to be a fact is entirely opposed to what we already know on the same subject, it is usually declared to be false. Thus, we would refuse to believe that some person whom we know well and respect was guilty of theft; for it would be impossible to connect such conduct with what we already know of his character. And, similarly, we find it impossible to believe, even although we have the evidence of our senses, that the conjurer has actually performed what he professes; for to do so would often be to reverse entirely our conception of natural laws. It must not be forgotten, however, that the existing system of knowledge, which seems to serve as the standard and test of new facts, is itself undergoing constant modification through the influence of these facts. As new experiences are brought into connection with the existing body of our knowledge, there is a constant rearrangement and readjustment of the latter going on. Usually this adjustment is slight, and takes place almost imperceptibly. But, in some

cases, a single fact may be so significant as completely to transform what seemed to be the accumulated knowledge of years. The experiment which Galileo made by dropping balls of different weight from the tower of Pisa, made it impossible to hold any longer the old theory — which seemed as certain as anything well could be — that the velocity with which bodies fall is proportional to their weight. Again, if theft were actually proved against the man we respect, that single fact might be sufficient to force us to give up everything which we supposed that we knew about his character.

(2) We have said that judgment is the process by which knowledge grows into a system. It is by judging or thinking that we attempt to bring the various parts of our experience into relation with one another. The degree to which this has been done is the measure of our intellectual development. The knowledge of the uneducated and unthinking man, like that of the child, is largely composed of unrelated fragments. It is an aggregation, not a system of facts. The facts which go to make it up may quite well be contradictory, but this contradiction is not seen because no attempt is made to unite them. There is, of course, no human experience which is entirely systematic, or which has been completely unified. Even those who have thought most deeply find it impossible to fit together exactly knowledge gained from different fields, and from different sciences. The facts of one science, for example, may seem to stand by themselves, and not to have any relation to the facts derived from another science. Or there may appear to be a conflict between the results of physical sciences, and the truths of moral philosophy and religion. But the ideal always remains that truth is one and indivisible, and that it must be possible ultimately to harmonize all facts in one all-embracing system of judgment.

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CHAPTER XXII

THE LAWS OF THOUGHT

§ 79. **The Law of Identity.** — We found (§ 73) that Judgment is the simplest form of thinking. And, in the last chapter, we were engaged in studying its main characteristics, and becoming acquainted with its mode of operation. The essential nature of the thinking process, therefore, has already been stated, though we have not traced the mode of its development, nor shown its application to the various problems of experience. In nearly all books dealing with logic, however, one finds a statement of three fundamental laws of thought which differ greatly, in form at least, from what we have so far learned regarding the nature of Judgment. These laws are so well known by name, and yet so ambiguous in their mode of statement, that it seems well to try to decide what meaning to apply to them. It will also be interesting to note their relation to the discussion of Judgment already given. They are usually regarded as axioms, or propositions which require no proof, rather than as descriptive of the nature of thought. In this sense, they are supposed to be the foundation of all logic, since they are presupposed in all thinking.

The first of these laws, or axiomatic principles, is that of Identity. Whatever is, is; everything remains identical with itself; A is A. These are some of the forms in which the law is usually stated. In all argument, we

assume at least that each thing possesses a permanent character, and does not pass now into this, now into that. If any knowledge is to be possible at all, the character of things must remain fixed. Socrates is always to be Socrates, and iron, iron. Every one assumes as much as this, though he may not himself be conscious of it (cf. § 9).

Another interpretation of this principle was, however, offered by Boole and Jevons, who developed what is known as the Equational, or Symbolic logic. According to these writers, the law of Identity expresses the fundamental nature of Judgment. That is to say, every judgment is the expression of an identity between the subject and the predicate. The judgment, 'New York is the largest city in America,' is simply a case of *a* is *a*. It expresses the fact, that is, that New York and the largest city in America are identical. 'Iron is a metal,' is another example of the same principle. It may be written: iron = metal. And, since the copula may often be ambiguous, it will be better to discard it in working out arguments, and adopt, in its place, the sign of equality.

Judgment, then, is simply an equation, and may be written as such. Further, the conclusion of a series of logical premises may be obtained by a process similar to that employed in working algebraical equations. That is, we can substitute for any term in a judgment, its equivalent, or the value which it has in another judgment. This method Jevons calls 'the substitution of similars,' which he maintains is the fundamental principle of all reasoning.

If, now, we employ letters to symbolize the terms of the propositions, it is claimed that we can work out any argument by the equational method. Take the argument,

All metals are elements,
Iron is a metal,
Therefore iron is an element.

Now represent metal by M; iron by I; and element by E. Then the argument in equational form will be,

$$M = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$I = M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and by the substitution in (1) of the value of M in (2) we get $I = E$, the required conclusion.

Or, we may illustrate this method by a somewhat more complex example which is also taken from Jevons: 'Common salt is sodium chloride, which is a substance that crystallizes in cubical form; but what crystallizes in cubical form does not possess the power of double refraction.' The conclusion of this argument may be found by letting A = Common Salt, B = Sodium Chloride, C = something which crystallizes in cubical form, and D = something which possesses the power of double refraction. The negative of any of these terms will be expressed by the corresponding small letters. The argument may now be expressed:—

$$A = B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$B = C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$C = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By substitution of the value of C in (2) we get,

$$B = d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

And substituting here the value of B in (1),

$$A = d$$

Giving to these symbols their meanings, we get the result 'common salt does not possess the power of double refraction,' which is the conclusion of the argument.

Of course, in simple arguments like those we have been examining, there is nothing gained by the use of symbols, and the representation of arguments in this form. But when the various terms employed are much longer and more complex, simplification may be attained in this way. Various other symbols have also been used to express the relation of the various terms to each other, and a symbolic logic has been developed which follows very closely the procedure of algebra. The examples given may, however, serve as illustrations of this method.[1]

It is, however, as a *theory* of the meaning of Judgment that we are interested in this mode of interpreting the law of Identity. We have seen that it works fairly well in practice, and therefore cannot be wholly false. But there are certain forms of reasoning in which it will not work. We cannot get the conclusion by the equational method in an example like the following: 'B is greater than A, C is greater than B, therefore C is still greater than A.'

This practical objection being left out of account, we have to ask whether an equation represents fairly the nature of Judgment. Does a judgment express merely

[1] The clearest statement of the aims and methods of the Equational Logic may perhaps be obtained from Jevons, *The Principles of Science*, Introduction. Cf. also G. Boole, *An Investigation of the Laws of Thought*. London, 1854.

the identity of subject and predicate? And if so, what *kind* of identity is referred to? In mathematical reasoning, the sign of equality expresses the identity of quantitative units. When one says, $2 + 3 = 5$, the meaning is that the number of units on each side of the equation is identical. And, similarly, the assertion that a parallelogram = 2 triangles with the same base and of the same altitude as itself, expresses the fact that, in the two cases, the number of units of area, square feet, square yards, etc. is the same. In mathematics, the equation declares that the quantitative relations of its two sides are identical. It does not assert that the two things compared—the triangle and one half the parallelogram, for example—have the same qualities, or are exactly the same in all respects. Now, if we extend the use of the sign of equality, it must take on a new meaning. It is clear that in a judgment like 'iron = metal,' there is no reference at all to quantitative relations. We are not asserting that the number of units in the two terms is identical. What, then, does the sign of equality express in such a case?

The answer is not difficult, say those who hold this theory. The sign of equality in such cases expresses *absolute identity;* the entire and complete sameness of subject and predicate. The proposition, 'mammals = vertebrates,' asserts that mammals and vertebrates are one and the same thing. But that statement in its present form is not true: the class mammal does not completely correspond with the class vertebrate. To make it exact, say those who uphold the equational form, one must qualify or limit the predicate and write

the proposition, 'mammals = some vertebrates.' But, even so, we may urge, the form of the judgment is still defective. In the first place, it does not correspond to the model $a = a$. For one side, 'mammal,' is clearly marked off, while the other is indefinite and vague. And, secondly, just because of its vagueness, it is not a satisfactory piece of knowledge. To obviate these objections, one must go further and write, mammals = mammalian vertebrates. At last the judgment seems to correspond to the type, $a = a$. But a new difficulty arises. Has not the judgment lost all its original meaning and become a mere tautology? There seems to be no escape from the following dilemma: *either* there is some difference between subject and predicate, and the judgment is therefore not in the form $a = a$, *or* the judgment is tautologous and expresses nothing. The view of the equational logic that Judgment affirms the entire identity of subject and predicate refutes itself. The form $a = a$ cannot be regarded as the type to which all judgments conform.

But there must be some kind of identity between the parts of a judgment. In one sense, we do seem to declare that the subject and predicate are identical when we say, 'iron is a metal.' As we have seen, however, if these terms are merely identical and nothing more, the judgment loses all meaning. We are forced to the conclusion that every judgment affirms both identity and difference, or that there is identity running through and underlying the diversity. But is not this a paradoxical statement? When we affirm identity, does not this imply the absence of all difference? If

a is *a*, how can it at the same time be something different from itself?

And yet this is just what every judgment which has any meaning affirms. 'Iron is fusible.' 'This table is made of oak.' 'The sword is rusty with age.' In all these judgments there is an assertion of the unity of different properties or parts in one whole. A is B, and yet does not cease to be A, is rather the type of judgment than *a* is merely or abstractly *a*. It is worth noticing that this view of the matter corresponds with the account of Judgment already given. We saw that Judgment constructs a *system* of knowledge by showing that various things, which seem at first unrelated, are yet connected by an underlying unity. Knowledge is always the synthesis or union of different parts or different properties in a common identity. And each judgment, as an element of knowledge, displays the same essential structure which belongs to knowledge as a whole. It involves, as was shown in (§ 77), both analysis and synthesis, and declares the oneness or identity of a number of properties or parts, without at the same time losing sight of their distinctness.

Let us now sum up our discussion of the law of Identity. When rightly understood, as we have seen, it does not affirm that *a* can only be bare *a*, that the subject and predicate are absolutely identical. It is a law of thought, and expresses the fact that Judgment brings together differences; *i.e.*, different things and qualities, and shows that they are parts of one whole or unity. It reveals the underlying unity or identity which is present in the midst of variety. This law also states

another characteristic of Judgment which we have already emphasized. This is what we have called the universality of Judgment (§ 75). It is to judgments, and not to concepts or terms, as has sometimes been supposed, that the law of Identity properly applies. What it affirms in this connection is simply that Judgment claims to be true, and hence is identical at all times and for all persons. It cannot be true for you and false for me that, 'iron is a metal.' Truth is not a matter of individual taste, but every judgment which is true has a permanent character or identity belonging to it.

§ 80. **The Law of Contradiction.** — The law of Contradiction is the second of the so-called laws of thought. It is usually stated as follows: It is impossible for the same thing both to be *a*, and not to be *a*; or, *a* is not *not-a*. It is evident that this law states in a negative form the same characteristics of thought as the law of identity. Indeed, it was in this form that the principle was first laid down by Aristotle. "It is impossible," he says, "that the same predicate can both belong and not belong to the same subject at the same time, and in the same sense."[1] We cannot assert in the same sense that Socrates is both wise, and not wise. Truth is not, as the Sophists supposed, a matter of taste or convenience, but must be consistent with itself. If a judgment affirms that 'iron is a metal,' it at the same

[1] *Metaphysics*, Bk. III. Ch. IV. See also the remaining chapters of the same book for Aristotle's demonstration that all thought presupposes such a principle.

time excludes the assertion that it is not a metal. There is a fixity and permanence about judgments which prevents them from changing into anything else. And it is just this permanence which we have already called the universality of Judgment, which the law of Contradiction expresses in a negative form.

The law of Contradiction has, however, sometimes been interpreted in such a way as to make it equivalent to the assertion of abstract or bare identity which we found in the Equational logic. That is, the statement that it is impossible for any judgment to unite *a* and *not-a* may be taken to mean that it is impossible to assert the unity of *a and anything different from a*. But, as we have seen, this is exactly what we do in every judgment which is more than a tautology. The law, then, does not forbid the union of *differences* in one judgment, but of *contradictories*, or of what would destroy the integrity of the judgment and render it unmeaning. If the law is to hold true of Judgment, *not-a* must not be taken as equivalent to anything which is different from *a*, but as signifying what is opposed, or contradictory to *a*.

It is not by any means easy to decide what things are merely different, and therefore compatible with each other, and what contradictory or opposed. Logic can give no rule which may be applied in every case. If experience shows that two things, or two properties, are at any time united, we say that they are merely different from each other; if they have never been found in conjunction and we are not able to conceive how their union could take place, we call them opposites or contradictories. It is worth noticing, too, that no terms are *in themselves* contradictory, except those which are in the form *a* and *not-a*, wise and not-wise. But they *become*

contradictory and exclude each other when they claim to occupy the same place in some particular system of facts. Thus 'maple' and 'oak' denote trees of a different variety, which are, however, so little opposed that they may exist side by side. If both these terms were applied to the same tree, however, they would become contradictory. By claiming to stand in the same relations, these terms become rivals, as it were, and exclude each other. But a knowledge of the particular facts involved is always necessary in order to determine whether or not two assertions are really incompatible.

§ 81. **The Law of Excluded Middle.** — The third law is a corollary from what has just been said in the last section. There is no middle ground, it declares, between contradictories. A is *either b or not-b*. To affirm the one is to deny the other. When we have real contradictories, — *i.e.*, when *not-b* is not merely something different from *b*, but something which excludes it, — every judgment is double-edged, and both affirms and denies at the same time. To deny that the throw of a penny has given heads, is to assert that it has fallen tails. As we have seen, however, logic affords no rules of deciding when things do thus stand in the relation of mutual opposition. The law of Excluded Middle states only that *where* this relation does exist, every proposition has a double value, and both affirms and denies at the same time. It requires special knowledge of the particular facts in each case to enable us to decide what things are thus opposed to one another. There is no logical law by means of which things may be divided into two opposing groups or classes.

It is important to notice that all of the judgments which we use in everyday life are to some extent double-edged. That is, they contain, besides what is directly affirmed, some implication or counter statement. For example, to say, 'that object is red,' is implicitly to deny that it is blue, or any other colour. The statement, 'A never looks at a book,' carries with it the implication that A is not very intelligent. In almost any field where we have any systematic knowledge, we can limit pretty definitely the number of possibilities — *a* must be either *b*, or *c*, or *d*. In such cases, to affirm that *a* is *b*, is of course to deny implicitly *c* and *d;* and conversely, the denial of any one possibility, as *c*, enables one to assert that *a* is *b* or *d*. In ordinary conversation, misunderstandings and misconceptions frequently arise because neither party is fully aware of all the possible cases and the relation between them. It is very difficult, however, to make a statement which will have no counter implications. If one says, 'this railway system does not employ steam power,' the proposition seems to justify the question: 'Does it then use electricity or compressed air?' We should feel that it was a mere quibble if the person who made the statement should reply: 'I did not say that it employed any kind of power.' 'There are some small errors in this paper,' would ordinarily be taken to imply the counter proposition, 'the paper contains no serious errors.' It is clear that it is only when one's knowledge becomes *systematic*, — *i.e.*, when one knows the relations in which all the facts in the field under consideration stand to each other, — that one can be fully aware

of what is really implied in each assertion or denial (cf. §§ 41, 78).

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CHAPTER XXIII

TYPES OF JUDGMENT

§ 82. **Judgments of Quality.** — We have hitherto been considering the nature of Judgment in general, and have learned something regarding its main characteristics. It is now necessary to examine briefly some of the more important forms or types of Judgment. We shall begin with very simple and elementary ways of judging, and afterwards consider some of the more complex types. In this way, we shall see the nature and structure of Judgment illustrated at different levels of thought. And we also hope to show that there are no arbitrary divisions in the process of thinking, that the lower forms of Judgment gradually develop into the higher in accordance with the general law of evolution. It is, of course, impossible to carry out at present this plan in detail, for that would be to give a complete history of the development of thought. It will be necessary for us to take long steps, and content ourselves with a general view of the relation of the various stages in the development of Judgment.

The first efforts of intelligence to understand the world take the form of judgments of Quality. At a low stage of mental development, it is the simple qualities of things which force themselves on attention. The young child, for example, takes notice of only the

most striking qualities of things. His judgments are very vague and indefinite, and take account only of some prominent quality of things. That is, there is no discrimination of the various parts and relations of the objects, but the judgments express merely a general impression based upon some striking quality. Thus it has often been noticed that the child calls every man 'papa,' and any light, of whatever size, the moon. A little boy, known to the author, used to call Sisters of Charity, crows, on account of the colour of their dresses. The objects as he apprehended them were simply black, and nothing more. His intelligence rested in the qualitative total impression; the various parts, with their quantitative relations, which he afterwards learned to know and distinguish, did not at that time exist for him.

It is perhaps impossible to find in the experience of an adult any judgments which deal entirely with simple qualities, and which take no account of the numbers, and even to some extent of the relations, of the parts. But we can find examples of judgment where the qualitative aspect is much the most prominent — where indeed the quantitative and more complex relations are scarcely noticed at all. 'This is green,' 'that is a strange odour,' 'there is something a long way off,' — all these seem to be judgments of quality or general impression, and to involve scarcely any other element. It is, too, the easiest kind of judgment to make, the judgment which involves least mental effort, and which notices only the most evident, and, as it may be seen, the most superficial, aspect of things. It is evident that such

judgments belong to a lower stage of thinking, than those which imply analysis and perception of quantitative relations. Compare, for example, 'this is very large,' with, 'this object is made up of roots, trunk, branches, and leaves'; or 'this is green,' with, 'this leaf is divided into two parts by a rib running through the centre.' The first judgment in each pair obviously involves much less intellectual work than the latter. The judgment of simple quality is, as we have seen, the starting-point of thought. It is with this kind of thinking that the knowledge of the child begins. And, before the savage learns to count, *i.e.*, to distinguish and enumerate the parts of the objects with which he deals, his judgments must necessarily belong to this same type.

It must never be forgotten, however, that simple judgments of quality are really judgments; *i.e.*, are not given to the mind from any external source, but are the products of its own activity. A judgment, as we have already pointed out (§ 73), implies a reaction on the part of the mind on what is presented to consciousness through the senses. It distinguishes and puts together the material which sense presents in such a way as to perceive its significance — what it really amounts to — as a piece of knowledge. This act of interpretative intelligence has gone, however, but a little way in the type of judgment with which we are dealing. But even in a vague qualitative judgment like, 'there is something black,' the essential characteristics of Judgment can be already distinguished. For it presupposes at least some analysis or discrimination of the black object from the

rest of the environment, and of the black colour from other colours. And the judgment, 'something is black,' has made at the same time a beginning in constructing this vague something into a system of qualities, or into a thing that is known. The other qualities and relations are as yet wrapped up in the indefiniteness of the 'something.' In spite of its indefiniteness, however, the latter plays the part of a permanent centre or identity. It is the whole from which the quality of blackness has been separated out, and to which it is again attached.

Our thought, however, is not satisfied with a knowledge of the general qualities of things, but pushes farther its work of analysis and construction. In this way, it begins to distinguish the various parts of objects, and to compare one with another. We not only judge that 'the grass is green,' but go further and say 'this piece is dark green, and that light green.' The indefinite judgment, 'this cane is heavy,' is no longer satisfactory, and is replaced by, 'this end of the cane is much heavier than that.' And when this stage is reached, judgments of Quality are already passing into the next higher type, judgments of Quantity. For the moment of comparison, which is already contained in these judgments, is the basis of counting, measuring, and all quantitative determination. In advancing from the simple apprehension of quality, to take note of, and compare, the *degree* or *intensity* which the same quality manifests in different instances, intelligence has entered upon a path which leads directly to judgments of quantity. To distinguish parts, to regard things as degrees or instances of a common quality, is at once

to suggest the quantitative process of counting and measurement.

§ 83. **Judgments of Quantity.** — It is very difficult, as we have seen, to draw a hard and fast line between quality and quantity. Indefinite judgments of general impression which do not imply any comparison, seem always to be qualitative rather than quantitative in character. This is true, I think, of judgments like, 'this object is very large,' 'there was a great flock of sheep in the field.' In such cases, the interest does not seem to be quantitative at all; *i.e.*, there is no effort made to determine *how many* units or parts there are in the whole about which the judgment is made. But the general impression of size or number is apprehended and judged of at the same level of intelligence, and in the same vague way, as the simple qualities with which we dealt in the last section. It is by means of such a general qualitative impression that the savage who cannot count beyond five, is able to distinguish between six and some larger number. And we must suppose that the shepherd's dog does not learn that some of the sheep are missing by any process of counting. We must suppose that the general qualitative impression made by the smaller flock is different from that made by the larger, and that there has been no real counting or estimation of number in the case.

But quantitative judgments proper belong to a higher stage of intelligence than do those which have just been described. Indefinite judgments, like 'this is very large,' or, 'there are a great many stars in that group,'

are not satisfactory pieces of knowledge. We accordingly set ourselves to get more exact information about the parts which compose the wholes. The first step in this process leads to *Judgments of Enumeration.* If the whole which is analyzed is composed of homogeneous parts, the judgments of enumeration take the form of simple counting. 'There are one, two, three, . . . twenty men in this company.' Where the parts are not of the same kind, however, a separate name may have to be given to each. 'This plant is composed of root, stalk, leaves, and flower.'

But exact quantitative knowledge requires us to do more than enumerate the parts of which a whole is composed. We must go on and *weigh* or *measure* them. There is of course no essential difference between weighing and measuring, so that we may call all judgments which express the result of this process *Judgments of Measure.* It is worth noting that judgments of this class are not so simple and direct as may appear at first sight. When we measure, we express the relation of the parts with which we are dealing to some common unit or standard. The judgment, 'this tower is 200 feet high,' means that if the tower is compared with a foot-rule, it will be found to contain it 200 times. It really, then, involves a proportion, and might be expressed :– tower : foot-rule = 200 : 1.

The point which it is important to notice is that all measurement is the result of comparison. In the first place, some unit is more or less arbitrarily selected. Then the judgment states simply the relation between this unit and the object measured : one is contained in

the other once, or twice, or ten times. The quantitative determination thus obtained, then, is merely relative. That is, it does not belong absolutely, and in its own right to the object measured, *but indicates the relation of that object to something else.*

For this reason, it may seem that quantitative relations tell us nothing regarding the real nature of objects, and that to discover what the latter are *in themselves*, we shall have to return to the point of view of quality. But we have seen that simple judgments of quality yield a very unsatisfactory kind of knowledge. Moreover, we should find on examination that even qualities always imply a reference to each other, and are no more absolute than quantities.

In order to obtain more satisfactory knowledge regarding things, we shall have to go forward to a higher type of judgment, rather than backward to quality. But the importance of quantitative determination for exact knowledge must not be overlooked. By means of measurement, things are reduced to common terms, as it were, and thus a basis of comparison is afforded where it would otherwise be impossible. To reduce everything to such a common measure is the business of the physico-mathematical sciences. Everything has a quantitative value, and can be expressed mathematically in terms of some unit or standard, as, for example, the unit of heat, or of pressure, or the electrical unit. It was this tendency to count and measure and weigh things which established the body of exact knowledge which we call science. And in almost every field, knowledge increases greatly, both in extent and exact-

ness, as soon as it is found possible to reduce all phenomena to a common measure, and to express their relations by means of mathematical formulas.

It is a great step in advance to be able to compare things as quantities, and to express their relations in terms of number. But judgments of quantity are not entirely satisfactory; they are, as has already been noticed, merely *relative* in character. Moreover, from a quantitative point of view, each thing is equivalent to the sum of its parts. When the parts have been enumerated and measured, the value of the whole is obtained by addition. But it is scarcely ever possible to represent adequately the nature of a whole in this way. So long as we are dealing with a piece of inorganic matter, the method of regarding the sum of the parts as equivalent to the thing, generally gives good results and leads to no difficulty. But it is quite different when the whole in question belongs to something which has life and consciousness. In such cases, we have what has already been called an organic whole (§ 78). Now, it is clear that the principle of quantity, which can only add and subtract, is insufficient to represent completely the nature of an object of this kind. It has no means of representing the individuality or real whole, which rather constitutes the parts, than is constituted by them. That is, to understand such objects, we shall have to take a new point of view, and begin with the whole rather than with the parts. From the point of view of quantity, the nature of the whole is discovered by adding together the parts; while in order to understand objects which possess an individuality of their own, there seems to be a central principle to which the parts are subordinated, and in relation to which alone they can be understood. The type of judgments which deal with such objects we shall have to discuss in § 85.

§ 84. **Judgments of Causal Connection.** — Another class of judgments used in building up knowledge, may be called judgments of Causal Connection. They undertake to show how the various changes which go on in

things are connected causally with other things or events. This type of judgment — leading as it does beyond the particular object, to a knowledge of the ways in which objects are connected — seems to belong to a higher stage of mental development than those which merely take note of quality and quantity. This does not mean that we never look for causes, until the qualities and quantities of things have been discovered. Nor is it true that any causal judgment, however vague and unsatisfactory, is higher than any judgment of quality or quantity whatsoever. But, in the beginnings of knowledge, one may say, thought does not travel outside the particular object to show the connections of the latter with anything else. And beginning in this way, it seizes first upon quality and quantity which seem to belong to things in themselves. We have seen, however, that as a matter of fact judgments of quantity involve comparison, and so a reference of one thing to another, though that reference is not usually made consciously or explicitly. But, when we judge that one thing is causally connected with another, the external reference has become explicit, and is the very essence of the judgment.

The word 'cause' has been used in a great many senses, and its various meanings have given rise to a great deal of discussion. That every event must have a cause, was formerly regarded as an innate truth, or *a priori* proposition. We have seen, however, that we do not come into the world with any ready-made stock of knowledge. All knowledge, we have often repeated, is the result of the mind's own judging activity. The so-

called law of causation (every event must have a cause) must therefore express the fact that thought does connect things as causes and effects. Intelligence is not satisfied to take things in isolation; it tries to gain an insight into the ways in which they are connected, to discover what one has to do with another. And this is just the characteristic of thought which was emphasized in § 78. Judgment, it was there said, is a process of constructing a *system*, of showing how the various parts of knowledge fit into one another, and are mutually dependent upon one another. The tendency of thought to connect things causally, then, is the same as its tendency towards a system, which has now become more explicit and conscious of itself in this type of judgment than it was in quality and quantity.

It will be interesting to note some of the most important changes which take place in the principle of causal explanation at different stages in the development of knowledge. The child and the savage regard all changes and events which take place in the natural world, as due to the agency of living beings. These beings are represented as more or less similar to men, and as endowed with human passions and emotions. Thus we say that the earliest kind of explanation is essentially anthropomorphic. This word is derived from *ἄνθρωπος*, a man, and *μορφή*, shape or form, and hence is used to describe the way of representing either a spiritual being, as for example, the Deity, or natural forces like fire, wind, etc., in human form. It is probably true that at a very early stage in the development of both the individual and the race, every object is

supposed to have life. Or, perhaps, it would be truer to say that the young child (and the same would be true for the savage on a low plane of intelligence) has not yet made the distinction between animate and inanimate objects, but vaguely regards everything as like himself. This stage is usually known as *animism*, because each object is supposed to be endowed with a spirit, or *anima*.

Gradually, however, the distinction between animate and inanimate objects becomes clear. Accordingly, we find that at a somewhat more advanced stage the mode of explanation takes a different form, though it is still anthropomorphic. Physical objects are no longer regarded as living, but the changes in them are supposed to be due to the action of spirits, who are outside of the objects, but who use them to accomplish their purposes. These invisible spiritual agents, to whom all natural events are referred, have been variously named. It is clear, however, that the gods of mythology belong here, as well as the fairies, elfs, ghosts, and witches of the popular folk stories. It was a great advance when a Greek thinker, named Thales, came to the conclusion that it does not in any way explain natural events to refer them to the action of the gods. For, in the first place, to say that the gods cause this or that event, is to state something which we have no means of proving. And even if the assertion were true, it would not really explain anything. For it would not enable us to understand *how* the changes in question came about. It would tell nothing whatever regarding the actual steps in the

process itself. Thales saw this, and tried to give a *natural* explanation of the world, and all that goes on in it. He tried to build up a real system of knowledge by attempting to show how everything which has happened in the world has been connected with some natural cause. We know very little about the actual explanation of the world which Thales gave, except that he tried to derive everything from water. It is on account of the method which he adopted, rather than of what he actually performed, that he is regarded as the founder of science. Thales first showed, one may say, that knowledge means an insight into the ways in which the actual phenomena of the world are connected. We cannot unite into a system things so different in kind as spirits and natural phenomena. Or we may say that real explanation demands that there shall be some likeness, or ground of similarity, between the cause and the effect. An event which happens in the world of objects, must be explained by showing its connection with some other event, of a similar character, which precedes it.

The development of this conception of scientific explanation also influenced still further the notion of causality. We have seen that in the beginnings of knowledge every event was supposed to be due to the action of some living agent, or spiritual being. Even after this mythological mode of explanation is discarded, and natural causes put in the place of spirits, it is still difficult to rid oneself entirely of the old anthropomorphism. The popular mind still tends to regard the cause as an *agent* which *produces* the effect, through some power or efficiency which it possesses. It

is not necessary to raise the question at present whether there are any grounds for this belief. To discuss this problem would carry us beyond logic into metaphysics. What we wish to notice is that science has gradually abandoned the notion that the cause *does something* to the effect. That, as we have seen, is a remnant of the old prescientific idea, and a notion which does not aid at all in explaining our knowledge. It is the business of science to show *how* the things and events which make up our experience are necessarily connected with one another. Science has to discover what things invariably go along with one another, and necessarily presuppose one another. And, when it is found that some particular thing or event, A, invariably precedes another particular occurrence, B, the former is regarded as the cause, and the latter as the effect. In order to eliminate as far as possible the notion of agency or efficiency which attaches to the word cause, the terms 'antecedent' and 'consequent' are often used to indicate this relation. For science, the cause is not an active agent, but the invariable antecedent of something else which simply follows it. The cause does not explain the effect by assigning an agent which brings the latter about through its personal efforts; but it explains, because it reveals another necessary step in the process, and gives us a new fact which joins on or can be connected with the one from which we start.

We conclude then that the cause of any event is its invariable and necessary antecedent. In another part of this book (Chs. XV., XVI.), it is shown what tests it is necessary to apply in order to determine whether two

phenomena are merely accidentally conjoined, or whether the connection is essential and real. It is necessary now to take one more step in tracing the various ways in which the idea of causality has been used. As a result of a famous scientific discovery, which was made about the middle of the present century, a new element has been added to the notion of cause in its application to physical phenomena. The law of the Conservation of Energy states that the amount of energy, or power of doing work, possessed by any set of bodies, remains constant. Any change in a material body is the result of a transformation of energy from one form to another. The same is true of the world as a whole: the total amount of energy which it contains remains constant. All changes which take place in the physical universe — motion into heat, or electricity into motion — are simply different forms, or manifestations, of the one world-energy.

As a result of this law, the effect always represents the same amount of energy, or power of doing work, as the cause. Since no energy is ever lost, the one must be equal to the other. And, as a matter of fact, the quantitative equivalence of many of the various forms of energy has been proved by actual measurement. In working out this law, for example, Joule showed that "the energy stored up in the 1 lb. weight which had been pulled up 772 feet was gradually transformed, as soon as the weight was released, into an amount of heat capable of raising the temperature of a pound of water 1° Fahr.; while Hirn showed, on the other hand, that exactly this amount of heat would, if it could be turned

back again into energy, raise the 1 lb. weight to the height of 772 feet at which it stood before."[1]

The new element which this law adds to the idea of cause as a necessary and invariable antecedent, is that of the quantitative identity of cause and effect. Taking the phenomena which are connected in this way to represent simply certain quantities of energy, we say that the one is equivalent to the other. The energy which the cause represents has been transformed without loss, and reappears in the effect. If what seems to be the total effect is not equal to the cause, part of the energy of the latter must have been transformed into something else. No energy can have been lost.

It becomes, therefore, the task of the physical sciences to show that this relation of quantitative identity exists between phenomena which are causally connected. The ideal of physical science, is to prove that two phenomena are connected as cause and effect, by showing that both represent the same quantity of energy. For this purpose, measurement and calculation are necessary. The physical sciences, as was pointed out in the last section, deal largely with judgments of quantity, and devote themselves to showing by measurement that the same amount of energy persists through the various changes which phenomena undergo. In establishing causal connections, the physical sciences find it necessary to use the principles of measurement and calculation.

It will be evident, from what has been already stated, that this relation of cause and effect should apply to all phenomena whose

[1] Buckley, *Short History of Natural Science*, p. 339.

energy is capable of being measured and represented in quantitative terms. As a matter of fact, however, the law has been proved only in physics and chemistry. From the very nature of the case, it is extremely difficult to measure exactly the relations of cause and effect in the sciences which deal with organic life. But even in those sciences, the law of the Conservation of Energy is assumed to hold true. For example, the amount of energy which a plant contains, is assumed to be exactly the same as that represented by the various elements or forces — water, sunlight, mineral substances, etc. — which were instrumental in composing it. In the same way, we suppose that the same relation holds of the changes which go on in the brain, though we are, of course, unable to prove this by actual measurement.

It is difficult, however, to see how this law can have any application to mental phenomena. We can indeed measure the intensity and duration of sensations. But neither feelings nor complex processes of mind seem to be capable of measurement. Moreover, it is never possible to measure the energy, or power of doing work, which states of consciousness possess, and to equate one with another in this respect. And this being so, the law of the Conservation of Energy cannot, of course, apply to psychical causes and effects. In the mental sciences, then, we cannot claim that the notion of Causality contains the element of quantitative identity between cause and effect which has been found to exist in the physical sciences.[1]

§ 85. **Judgments of Individuality.** — By Judgments of Individuality, we mean judgments which regard some complex object as a real whole with a definite nature of its own. We have already had occasion (§ 78) to distinguish a mere aggregate or sum of parts, like a heap of stones, from a true whole which possesses a certain character and individuality of its own. It is the former point of view from which judgments of quantity and

[1] Cf. Wundt, *Ethik* (1st ed.) pp. 398 f.; Sigwart, *Logic*, § 97 *a*, 7.

of causal connection regard objects. For these types of judgments are concerned wholly with the parts — the former to measure, and the latter to show their causal connection. It requires a new form of judgment to represent adequately the nature of a complex object which possesses individuality. This form gives expression to the organic unity and wholeness of things, and emphasizes the way in which the parts coöperate for a common purpose or end. Thus we regard the parts of a plant as a unity coöperating in a common purpose, and a man as a conscious system of ends.

(1) We have seen that judgments of causal connection relate phenomena as causes and effects. A change in an object is explained by showing that some other change or event invariably precedes it. But this change, in its turn, demands explanation, and has to be accounted for by the discovery of a new cause. This type of judgment shows that one phenomenon is connected with a second, and a second with a third, and so on indefinitely. The view of the world which it presents is that of a never-ending series of causes and effects. It is never possible to find a cause which is not itself the effect of something else. No phenomenon possesses any independence of its own, but is simply a link in a series, or a piece of a whole that is never completed.

In the last section, it was stated that causal judgments connect one part of our knowledge with another, and, in this way, aid in uniting the parts of our experience in a systematic way. Now it is undoubtedly true that it would be impossible to have any real knowledge of anything as a whole, or an individual, without knowing the way in which the parts are related, and mutually depend upon each other. In that sense, judgments of causal relation are indispensable to a knowledge of a true whole. But this form of judgment itself resolutely goes on connecting part with part — one phenomenon with another — and refuses to regard any group of parts as possessed of an independent character or individuality

From this point of view, everything is externally determined; its cause, or principle of explanation, lies outside of it in something else. The mark of individuality, on the other hand, is the power of origination, or self-determination.

(2) Psychology, one may say, adopts the standpoint of Causal Connection; Ethics, that of Individuality. The former science regards mind as a *sum* of mental processes, and undertakes to show how its various parts are connected. Every state of consciousness is supposed to be determined by something external to itself — some antecedent mental state, or some bodily process. The interest, as was previously said, is centred in the parts, and it is very rarely that the psychologist stops to look at the mind as a whole. Ethics, on the other hand, has to begin with the individual. It does not regard mind as a thing or substance (that is the naïve point of view against which psychology rightly warns us), but as a self-conscious system of ideas, purposes, and feelings, which possesses the power of initiating action, and of determining itself. Ethics can adopt all that psychology has to tell regarding the mechanism of the mental processes. Indeed, without a systematic and detailed account of the nature and laws of mental life it could have no adequate conception of mind as a whole: the judgment of Individuality must use the results of judgments of Causal Connection. What it really does, is to transform the *sum* of mental processes into a *system* which has a real unity of its own. For it is only when a person is regarded as a self-conscious and self-acting individual, that he can be supposed capable of conduct to which the terms 'moral' and 'immoral' can properly be applied.

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CHAPTER XXIV

THE NATURE OF INFERENCE. — INDUCTION AND DEDUCTION

§ 86. **Judgment and Inference.** — It must not be forgotten that our object in these chapters is to obtain as definite a conception as possible regarding the nature of thought. To attain this end, we agreed (§ 73) that it would be advantageous to begin with the simplest or most elementary form of thinking. That form we found to be Judgment. We have now endeavoured to show what Judgment is, and what part it plays in building up knowledge. And, in the last chapter, we have attempted to see some of the steps in the evolution of Judgment, as it passes from simple judgments of Quality to judgments of Individuality. This account being completed, it remains now to discuss the nature of reasoning or Inference.

We shall probably get the clearest idea of the nature of Inference by regarding it as a completely developed judgment. As thinking develops from the form of simple judgment to that of Inference, it displays progressive differentiation and integration. In accordance with this law, we can say (1) that Inference is more complex than Judgment. The latter process, in its simplest form, can scarcely be said to have any parts: it represents a single act or pulsation of intelligence. Inference, on the other

hand, seems to imply steps or stages in thinking — a passage of the mind from one fact to another. Moreover, (2) Inference differs from Judgment in exhibiting the grounds upon which its statement rests. The simple judgment makes a declaration on the basis of sense-perception, as, for example, 'the mail-train has just gone down'; 'it rained yesterday.' Each of these statements stands alone, as it were; it does not attempt to gain support by pointing out the connection with other facts. To infer, however, is just to show the necessary connection of facts — that from the presence or absence of certain things, the presence or absence of certain other things necessarily follows. It is not necessary for Inference that the conclusion reached should be a fact which was not hitherto known. We often do reach new truths by reasoning from necessary connections. Thus we might *infer* that the mail-train has just gone down, from the fact that this train is always on time, and that it is now five minutes past the hour. Or, we might prove, to a person who doubted the correctness of our memory, that it rained yesterday, by pointing to other facts with which rain is necessarily connected. We might point to the muddy condition of the roads, the swollen streams, or, perhaps, might remind the person who questions the statement, that it was yesterday that A was out driving, and came home soaking. In this way, one tries to exhibit the *necessity* of the fact under consideration; and to do this is to infer.

In the actual process of knowledge, we more frequently go from a fact to its reasons, than in the opposite direction. The intelligence begins by accepting all

the connections as true and universal which it meets with in ordinary experience, or which are suggested to it in any way. It does not trouble itself at all about the grounds of its judgments, and thus the insufficient basis on which many of these stand is at first not evident. The child, for example, believes everything which it is told by its mother or nurse, or it may be, all the pleasant things which it imagines. Very often, too, the judgments of older persons are determined by their own wishes. The French peasant girl was sure that it was impossible for the Germans to take Paris. Another principle upon which both children and adults quite unconsciously proceed, is that the future must always resemble the past. The child assumes that the order of events each day will be the same, — that there will always be games after dinner, and visitors in the afternoon, because that has happened a number of times in the past. And one may have no better reason for believing that the sun will rise to-morrow, than the fact that it rose yesterday and to-day.

In these early, unreflective judgments, the ground or principle upon which they are based is, of course, not conscious at all. Each judgment is accepted by itself, and no questions are raised as to how it is known. But the development of intelligence may be regarded as a process of becoming conscious of the reasons which show the falsity of certain of our beliefs, and the necessity of others. The original judgment is not in reality so isolated and unrelated as it appeared; it contains implicitly its own reasons. But the validity of its procedure cannot be made manifest, until the reasons

for the statement made by the judgment are brought to light. In the development of knowledge, the judgment must expand so as to show the reasons which it necessarily presupposes. In itself, it is only a fragment of the complete statement, and it tries to complete itself by making clear the nature of the whole which it involves. It is not until the implicit reasons which every judgment contains are thus brought to consciousness, that it can be either proved or disproved. Taking the mere judgment by itself, it is only possible to place one man's assertion against another's denial. But proof or disproof of a proposition implies that reasons are given for or against it. If its connection with some fact, or set of facts, known to be true, becomes evident on reflection, the *felt* necessity which the judgment possesses (§ 76), is transformed into logical necessity. But, if no such connection can be found, or, if the judgment in question is seen to presuppose propositions which are themselves false, we must, of course, cease to regard it as valid.

When a judgment develops so as to become conscious of its reasons, it has already taken on the form of Inference. And, as we have already seen, this is the usual procedure of knowledge. We begin by believing without reason, or we assume that certain things are true, and try to find reasons for our belief. The conclusion, which is, of course, logically last, is usually first for us, and we set out from it to find the grounds, or the premises.

This way, however, of proceeding from conclusion to premises, or from a judgment to its reasons, implies that the mind is already aware of the distinction between false knowledge and true, and therefore that the

work of criticising and testing knowledge has already begun. The criticism of knowledge is probably forced upon the mind at first by the practical consequences of false judgments. So long as false judgments lead to no unpleasant results, they are likely to pass unnoticed, without any question being raised regarding the grounds by means of which they are supported. The child usually believes all that he is told, until he discovers that his credulity is making him a laughing-stock, or has led to the loss of some pleasure which he values. Sooner or later he learns that the ground upon which he has been unconsciously proceeding — somebody told me — is insufficient. In the same way, the natural tendency to regard all connections which we happen to find existing between events as universal and necessary, becomes more critical and discriminating. The child soon learns that the events of one day do not necessarily follow in the order of the day before, and that it is not always rainy on Fridays, and fine on Sundays. But, in order to discriminate between what is true and what is false, he is obliged to go beyond the facts themselves, and to become more or less clearly aware of the grounds assumed in each type of judgment. He is forced to include in the judgment the reasons by which it is supported. And, in this way, the distinction between valid and invalid principles of connection is gradually learned. Through experience, which is more or less dearly bought, we learn that we cannot depend upon hearsay, and also that many of the most obvious connections between events are not essential, and have no claim to be regarded as universal laws. It becomes

evident that it is necessary, in order to reach true principles of connection, to take a wider survey of the facts, and to push the process of analysis further than is done by our ordinary judgments of sense-perception. For example, we may at one time have supposed it to be a universal law that hot water will break glasses when poured into them. But as soon as we have experience of any instance or instances to the contrary, we see that there is no essential connection between hot water and broken glasses. It is necessary then to go behind the obvious facts of the case, in order to discover what is the real antecedent in the two cases. The two instances — where the glasses break, and where they do not — *seem* to be the same; and yet, since the result is different, there must be a difference which further analysis will bring to light. It is by penetrating behind the point of view of ordinary knowledge, that science endeavours to show how phenomena are really and essentially connected.

The judgments of ordinary adult life usually involve some consciousness of their grounds, and are therefore so far inferences. But in many cases of this kind it would be difficult for the individual to state explicitly the reasons for his judgment. The connection which he asserts may be guaranteed to his mind by some complex set of circumstances very difficult to formulate. Or it may rest upon some general similarity or analogy, which is so obviously insufficient that he hesitates to acknowledge that it is the only ground he has for judging. Thus one may be vaguely conscious that one's only reason for liking A is his resemblance to B. It may be impossible to say exactly in what points A resembles B; one may proceed on a vague general similarity. Or one may hesitate to make clear, even to oneself, that the only reason for disliking A is

because of some external resemblance — in name, or dress, or figure — to C, whom one dislikes.

§ 87. **The Nature of Inference.** — We have seen that it is difficult to draw any hard and fast line between Judgment and Inference. In general, however, we may be said to reason when we do not simply accept a fact on the basis of sense-perception or memory, but show that it necessarily follows from some other known fact or facts. Inference, then, requires (1) that certain data or premises should be accepted as already known; and (2) it implies an insight into the necessary connection of some new fact or set of facts with what we already know. Thus one is said to infer B, when one sees that it necessarily follows from some fact which is already known. It is not necessary for an inference that B should never have been in consciousness before. As we have seen in the last section, what we very often do in inference is to show the reasons or necessity of some fact which we have previously accepted without knowing why. No matter whether we go from premises to conclusion (from the reasons to the fact), or in the opposite direction, from the conclusion to the premises, we are said to infer whenever we find the ground for the existence of one fact in the nature of another fact. In the former case, we use words like 'therefore' and 'consequently,' to indicate the connection; and when the reasons are stated last, 'for' and 'because.' Whenever these conjunctions are used correctly, an inference has been made, and it is always useful in following a course of reasoning to make clear to ourselves precisely on what grounds it has been made.

Although Inference seems very simple and very natural, its procedure is much more puzzling, when looked at closely, than one would at first imagine. As we have seen, there is no Inference unless the result reached is different from the starting-point. But how are we ever justified in passing from a knowledge of one fact to another different from it? How can we ever pass from the known to the unknown? The Greeks, who loved to bring to light the paradoxes which so often underlie familiar facts, used to discuss this question. How is it possible for that which is unknown — external to the mind — to pass into the mind and get itself known? It was to solve this puzzle that Plato propounded the doctrine that all knowing is remembering.[1] Knowledge, he declares, is not increased by learning that of which we were altogether ignorant, but by a process of calling to mind or recollecting the knowledge which the soul possessed in a previous state of existence, but which was forgotten when it entered upon the conditions of the present life. It was therefore no longer necessary to suppose, according to Plato, that the mind performed the impossible feat of knowing what is external to itself, or that things previously unknown pass bodily into our minds, and thus become known.

Plato was undoubtedly right in protesting against the popular view that knowledge is received into the mind, as food is received into the stomach. Knowledge, as we have frequently seen, comes from within, not

[1] This is the theory upon which Wordsworth bases his "Ode on the Intimations of Immortality."

from without. But the apparent paradox of knowledge may be explained without adopting Plato's poetical notion of a previous state of existence. We may admit that the process of inference would be quite inexplicable, if it proceeded from one fact, A, to a knowledge of a second fact, B, which is totally different from the former. When we examine cases of inference, however, we find that there is always a certain amount of identity between the two ends of the process. The conclusion is always different, and yet not entirely different from the premises. Thus, from the propositions, 'all metals are elementary substances,' and 'gold is a metal,' one can infer that gold is an elementary substance. It is possible to connect 'gold' and 'elementary.' Here the identical link — what is called in formal logic the middle term — is 'metal.' It is possible to connect gold and elementary substance, because the former is at the same time a metal, which in its turn is an element. Of course, these conceptions — gold, metal, element — are not absolutely identical; it was pointed out in (§ 79) that propositions cannot be regarded as expressing mere identity without difference. But we can say that there is a common thread or element running through these notions, which furnishes the principle of connection. Where we cannot discover such a common nature, no inference can be made. Thus, for example, it would be impossible to draw any conclusion from the statements that 'it rained yesterday' and 'gold has been discovered in Alaska,' because there is no common element or connecting thread present which would lead us beyond the premises.

In formal arguments the middle term, or connecting link, is usually explicitly stated; but in the actual process of reasoning things out, it is frequently necessary to go in search of it. We may notice, for example, that the fire in a stove burns more slowly when the damper is shut. In order to understand the fact, we have to find out some fact which is common to 'closed-damper' and 'slow-burning,' some link of identity, as it were, which enables us to pass from the one to the other. Such a connecting link is afforded, of course, in this case by the supply of oxygen. Darwin was noted for his keenness in detecting connections which escaped the ordinary eye, as well as for his skill in giving explanations of them. On one occasion, he observed that in the part of the country where he lived, clover was abundant in those fields which were situated near villages, while the outlying fields were almost destitute of it. What now, he asked himself, is the connecting link between these facts? Some investigation of the matter convinced him that the two agencies which produced this result were mice and cats. The field mice destroy the clover by feeding upon its roots, but the cats go out from the villages into the fields near by and kill the mice.

We have seen that the passage from one fact to another in inference does not involve a transition to something wholly different from the starting-point. There is always some aspect or feature in which the premises are identical with the conclusion. And it is on the strength of this identity that a passage can be made from one to the other. The same fact may be expressed differently by saying that all inference takes place within a system, 'where the parts are so held together by a common nature that you can judge from some of them what the nature of the others must be.' Suppose you were given the leaf of a plant. If you had some systematic botanical knowledge, it would be possible to infer the species of plant to which the leaf belonged. That is, from

the nature of a part, the nature of the whole to which it belongs could be determined. The part represents the whole — in some sense contains it implicity. It is said that the great naturalist Cuvier could determine by examining a single tooth the nature of the animal to which it belonged. Let us suppose that the tooth were that of a ruminant animal. Now a zoölogist, who knows the characteristics of such an animal, could draw various inferences regarding the possessor of the tooth. He could conclude, for example, that the animal to which it once belonged must also have had cloven hoofs. A single piece or part, that is, would enable one who knows the system or common nature to which all the parts belong, to judge what the other parts are like.

The examples just given have referred to the possibility of an inference from one part of an organism to another. But, as we have already seen, the systematic connection which here exists between the parts, is more or less completely present whenever it is possible to infer at all. Inference pushes further the work of constructing a system begun by Judgment (§ 78). If each thing was known by itself, if the parts of our knowledge did not fall together into systems where each part to some extent determines the nature of the other parts, no inference would be possible. It is because the various pieces of our knowledge are never independent of each other, but form an organic whole, like the members of a living organism, that certain facts follow, as we say, from certain other facts. And it is of course true, that as our knowledge in any field becomes more completely organized, it is more possible to use it as a basis for in-

ference. The better we are able to put together in a systematic way the various facts which we have learned about geology, or astronomy, or the weather, the more *significant* each fact becomes. The geologist may be able to tell from the appearance of the cliffs what has taken place in a locality thousands of years ago. And, similarly, for the fisherman, the temperature, direction of the wind, its rising or falling, etc., are all *signs* from which he is able to infer, more or less correctly, the kind of weather which may be expected. A person who had no systematic knowledge in either of these fields, would, however, see nothing in the scarred rocks, or in the sudden change of the wind; he might notice the facts, but would not be able to use them as a basis of inference.

It is important to notice that what has just been said goes to confirm our previous statements regarding the increasing degree of integration which knowledge shows in the course of its development. The knowledge of the scientist differs from that of the ordinary man, not only in the greater number of facts which the former contains, but also, as we have seen, in the degree of integration or coherence which these facts possess. Inference, then, is simply a deep insight, based on definite knowledge, into the necessary connection of things. It is an act of thought which discovers the essential relations between things which at first sight appear to have no connection with each other. As has already been said, it is a reasoned judgment; *i.e.*, a judgment which has become conscious of the reasons for the connections which it affirms.

§ 88. **Induction and Deduction.** — It has been already pointed out that there are two directions in which inference or reasoning may proceed. We may begin with certain facts or principles which are already known,

or are assumed to be true, and proceed to show that some result necessarily follows from them. Thus we might infer that if the draughts of a stove are closed so that the supply of oxygen is lessened, the fire will burn slowly; or from the relative positions and revolutions of the planets, that an eclipse of the sun will take place on a specified day and hour. This method of reasoning is known as Deduction. It proceeds, as we have seen, from premises to conclusion. In the first part of this book, this form of reasoning has been treated at some length and its rules of procedure stated. At present, we need only notice that in deductive reasoning the particular case is always brought under some general law or principle, which is already known or assumed as true. Socrates is known to be mortal, because as a man he falls under the general law that all men are mortal; the closing of the draughts is a case of lessened supply of oxygen, and, therefore, in accordance with the general law, a case of slow burning. A deductive inference shows what are the results of the application of a general law to particular facts or instances. It proceeds downwards, as it were, from the general law to its consequences.

In Induction, on the contrary, the procedure is just the opposite of this. We begin with particular phenomena, and try to discover from them the law or principle which unites them. Certain facts are observed to happen together, and the problem is to find the ground or explanation of this connection. Inductive inference is thus a process of reading the general law out of the particular facts. It is an insight

into the nature of the whole or system, based upon a careful examination of the parts. 'Yesterday the smoke tended to fall to the ground, and it rained in the afternoon.' These two facts may simply be observed a number of times without any thought of their connection. But intelligence asks: Why should they happen in conjunction? And to answer this question, we must begin by analyzing the facts in our possession. When the smoke falls to the ground, the atmosphere must be lighter than usual; this is the case when it contains a great deal of moisture; but when the atmosphere is in this condition, it usually tends to discharge its moisture in the form of rain; therefore we have the general law which enables us to show that the behaviour of the smoke and the rain yesterday were not only accidentally *conjoined*, but essentially *connected*.

Deduction and Induction, then, are both forms of inference, but the starting-point and mode of procedure of the one is different from that of the other. Consequently, it is not unusual to speak of them as two *kinds* of reasoning which are quite distinct and independent of each other. It is, however, important to avoid this popular error, and to remember that the real process of inference is in each case the same. The essence of inference, as has been shown, consists in the fact that it exhibits the manner in which particular facts are connected together into a system or whole. And this end is achieved both by Deduction and Induction. In the former case, the general law of connection — what we may call the nature of the system within which the particulars fall — is known, and we argue from this as

to the nature and relations of the various parts which fall within it. We have the common thread which unites the various facts in our hand, and following it out are able to show its application in determining the nature of events which have not yet come within the range of our experience. Knowing the law of gravity, for example, one could infer deductively what momentum a ball weighing one pound must necessarily have after falling one hundred feet. It would not be necessary actually to measure the momentum of the falling body in this particular case, but it could be shown to be the necessary result of the general law. What the deductive inference shows to us, is the way in which a general principle or law of connection runs through a group of facts, and constitutes them a real or organic whole. The same insight is reached by inductive inference, although the starting-point is entirely different. As we have already seen, induction begins by observing that certain phenomena are frequently conjoined, and attempts to discover some law or principle which will make the fact of their connection intelligible.

It is usual to say that in induction we go from the particular facts to the general law. The following, however, would be a more correct form of statement: Before the inference, we observe that a number of phenomena occur together, but do not know whether this conjunction is necessary or not; or, if we assume that it is necessary, we do not understand why it should be so. As a result of the inductive inference, we gain an insight into the necessary connection of the observed phenomena, and also understand the principle according

to which the latter are united. What we really obtain through an inductive inference is not only a general law, but also a perception of its concrete application to particular phenomena. This being so, it is clear that Induction and Deduction are not two different kinds of inference. Inference always implies an effort on the part of the mind to see how phenomena are necessarily connected according to some general principle. And, in carrying out this purpose, the mind must begin with the knowledge which it already possesses. When the general law of connection is known, and the object is to discover the nature of some particular fact, the method of procedure is deductive. But, when the problem by which we are confronted is to read out of the facts of sense-perception the general law of their connection, the method of inference which must be employed is that of induction. But from whatever point we set out, and whatever may be the immediate object of the inference, the result is always the same — an insight into the necessary connection of facts according to some general principle.

It is not unusual to hear the remark made that modern science has been built up by the employment of the inductive method. This must not, however, be interpreted to mean that deductive inferences are not also used in the discovery of scientific truth. Science (which is simply another name for systematic knowledge) is the product of thinking, and thought, as we have seen, is not limited to any one mode of procedure. Thought aims at extending knowledge, and so long as it can find any link of connection, or guiding thread, it

is not limited to any one direction, or to any fixed mode of working. It is, of course, to be admitted — and this is what is true in the statement which we have quoted — that general laws cannot be discovered without an examination of particular facts, and that their validity must always be tested by comparison with the facts. But as soon as a general law is discovered in any field, it is always used as a principle from which to deduce new results. When it is possible to employ mathematics in the calculation of these results, it is usually possible to extend our knowledge of the subject much more rapidly than before. Thus physics and astronomy owe their rapid development to the application of mathematics. It must be remembered, however, that this presupposes a certain stage of advancement — a certain inductive stage, as it were — on the part of the science. But even in this earlier stage, we are constantly employing deduction, — reasoning out the results of certain guesses or suggestions to see if they hold true (cf. § 47). Both in ordinary life, and in scientific procedure, we may see, Induction and Deduction are constantly employed together.

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CHAPTER XXV

RATIONAL AND EMPIRICAL THEORIES

§ 89. **The Point of View of Rationalism.** — In the historical sketch of logic given in Chapter II., it was stated that there are two rival accounts of the nature of knowledge, and the methods by which it is attained (cf. § 8). The first of these theories is known as Rationalism, and has its best known historical representative in Descartes; while Empiricism, the opposing theory, is associated with the names of the great thinkers, Bacon and Locke. The doctrines of both these schools have been frequently modified, and the contrast between them is now no longer so pronounced as it was formerly. In spite of this fact, however, the division has always represented two schools of thought whose general relations to each other have remained comparatively constant. In general, too, it has been true that English thinkers have upheld Empiricism, while Rationalism has had its home on the Continent — at first in France, and later in Holland and Germany.

Rationalism regards mathematics as the type of all knowledge. Its essential characteristic consists in the fact that it undertakes to derive all knowledge from general principles. These principles have sometimes been regarded as innate (truths which are stamped upon the mind at birth), or it has been supposed that

they are in some way known before experience, and have a right to the title of *a priori* propositions (§ 76). Notwithstanding the various forms in which their theories have been expressed, all rationalistic thinkers agree in regarding the first principles upon which our knowledge is based, as upon a different plane from the facts of ordinary life. While the latter are known only by experience, and may be wholly or partially false, the former are described as principles which are in themselves necessary, truths the opposite of which is inconceivable, or sometimes as the axioms presupposed in all experience. These principles being accepted, the problem which lay before Rationalism was to show how all the facts of our experience necessarily follow from them, just as the various propositions of geometry follow from the definitions and axioms which are assumed as the starting-point. As a matter of fact, however, the famous Jewish thinker, Spinoza (1632–1677), was the only man who ever attempted to carry out Rationalism in this systematic form. In general, one may say that rationalistic thinkers have been mainly interested to show that the facts of the moral and religious experience are logically derivable from certain necessary first principles. It was questions like those regarding the existence of God, the immortality of the soul, and the freedom of the will, which the rationalists were anxious to put beyond dispute. And, as a consequence, not nearly the same amount of effort was devoted to showing how the other facts of experience could be similarly derived from general principles.

It will be at once clear, from what has been already

said, that the great instrument of knowledge from this standpoint must be reason. Very little attention is paid to perception, and the experience which it furnishes is not regarded as entitled to the name of knowledge. In order to know, in the true sense of the word, it is necessary to show the systematic connection of every fact with some fundamental first principle; and this, of course, can be done only by the employment of reasoning. Perception gives us only the bare facts; it is reason which enables us to trace the mutual connections, and derivation of these facts from some general law. The weakness of the rationalistic position does not consist in its insistence on the necessity of connecting the particular facts of experience with general laws or principles, but in the assumption upon which it proceeded that these principles could themselves be derived from some other source than experience. The result was that the rationalists employed themselves too exclusively in deducing facts from general propositions which were assumed to be true without sufficient criticism and examination. They saw clearly enough that mere perception without general principles can never give us knowledge, but they did not understand that it is impossible to separate the latter from the former, and to regard principles as existing in the mind prior to experience.

§ 90. **The Doctrine of Empiricism.** — Empiricism maintains that all knowledge is derived from experience; and by experience is understood the separate unconnected facts with which the mind is furnished in perception.

Empiricism refuses to admit that we possess any store of first principles or general truths which are native to the mind, or are obtained from any other source than experience. It is impossible for the mind to know anything of which it has had no perception. Moreover, the very fact that perception is made the standard of knowledge, led to the belief that the mind is something essentially passive, upon which ideas are impressed by external forces. Empiricism regards knowledge as the sum of the particular facts furnished to the mind through sense, not as a system which is the product of the mind's own activity. As a consequence, there results an entirely different theory of knowledge from that which we have given in this book. Ideas are supposed to be furnished to the mind by the channel of the senses, or are compounded from simpler elements which are supplied in this way. And when ideas become united by standing in juxtaposition, or being associated in some other way, the result is a judgment. In this account, the judging, or interpreting activity of the mind, which we have made the source of all knowledge, is wholly omitted. Indeed, one may say that empirical theories undertake to explain knowledge without reference to the mind and its mode of activity. Although all empirical thinkers do not deny the existence of the mind, yet none of them wish to go beyond the particular facts, and to call in its aid as a principle of explanation.

The same insistence upon particular facts, and avoidance of general principles, is characteristic of empirical theories of reasoning. All inference, it is

maintained, is based upon a perception of resemblance between individual cases. The general law, or principle, is nothing in itself but an abbreviated statement of the manner in which all the instances act which we have hitherto observed. The clearest statement of this theory is given by John Stuart Mill, from whose work on *Logic* the following passages are taken: "Now, all which man can observe are individual cases. From these all general truths must be drawn, and into these they may again be resolved; for a general truth is but an aggregate of particular truths, a comprehensive expression by means of which an indefinite number of individual facts are affirmed or denied at once. . . . From instances which we have observed, we feel warranted in concluding that what we found true in those instances holds in all similar ones, past, present, and future, however numerous they may be. . . . When, therefore, we conclude from the death of John and Thomas, and every other person we ever heard of in whose case the experiment had been fairly tried, that the Duke of Wellington is mortal like the rest, we may indeed pass through the generalization, All men are mortal, as an intermediate stage; but it is not in the latter half of the process, the descent from all men to the Duke of Wellington, that the *inference* resides. The inference is finished when we have asserted that all men are mortal. What remains to be performed afterwards is merely deciphering our own notes."[1] In other words, Mill maintains that all inference is based upon the perception of particular

[1] Mill, *Logic*, Bk. II. Ch. III. § 3.

cases. There is no such a thing as reasoning from general truths or principles. We may, indeed, arrive at such general truths by repeated experiences, and store them up as maxims in our memory; but they are not at all necessary for the process of inference, which may be said to be always inductive in character, since it sets out from a perception of individual cases. "Induction, properly so called, . . . may be defined as a generalization from experience. It consists in inferring from some individual instances in which a phenomenon is observed to occur, that it occurs in all instances of a certain class; namely, in all which *resemble* the former in what are regarded as the material circumstances."[1]

This account of the way in which inference proceeds undoubtedly contains much that is true. Nevertheless, it is not, I think, an adequate statement of the *nature* of inference. What one misses chiefly is some insistence upon the fact that it is only in virtue of some identical link, or common element, which is present in all the individual cases, that one is able to pass from one to another. On this point we must refer to what was said in the last chapter (§ 87). It will perhaps be possible to gain a clearer idea of what is true and what is false in this theory, by considering further Mill's doctrine, that it is possible to reason from one particular fact to another, without any reference to general truths.

§ 91. **Reasoning from Particulars to Particulars.** — "Not only may we reason from particulars to particulars, with-

[1] Mill, *Logic*, Bk. III. Ch. III. § 1.

out passing through generals, but we perpetually do so reason. All our earliest inferences are of this nature. From the first dawn of intelligence we draw inferences, but years elapse before we learn the use of general language. The child, who, having burned his fingers, avoids to thrust them again into the fire, has reasoned or inferred, though he never thought of the general maxim, Fire burns. He knows from memory that he has been burned, and on this evidence believes, when he sees a candle, that if he puts his finger into the flame of it, he will be burned again. He believes this in any case which happens to arise, but without looking in each instance beyond the present case. He is not generalizing; he is inferring a particular from particulars. . . . It is not only the village matron, who, when called to a consultation on the case of a neighbour's child, pronounces on the evil and its remedy on the recollection and authority of what she accounts the similar case of her Lucy. We all, when we have no definite maxims to steer by, guide ourselves in the same way."[1]

The doctrine as thus stated by Mill is the extreme opposite of Rationalism. Not only are all general propositions derived from observation of particular instances, but they play no part in the process of inference proper. All reasoning, according to this account, is based on the perception of resemblance between individual cases. No common nature or general principle ~~seems necessary~~ to unite the latter into a system.

Nevertheless, it must be confessed that Mill's state-

[1] Mill, *Logic*, Bk. II. Ch. III. § 3.

ment affords an excellent account of many of our ordinary inferences. We may accept it, however, as a *description of fact* without committing ourselves to the *theory* which it contains. That is, it will still be necessary to ask if inference is not, after all, based on the perception of some general law or principle, although it is not always possible to formulate the nature of the latter. It does not seem to me that the nature of the inference in the cases cited is completely described when it is said to be a passage from one particular case to another which resembles it. For it is necessary to look further, and to see what is *implied* in the fact that one case is perceived to resemble another. When the child perceives that the bright object before him resembles something which previously gave him pain, he has got beyond the merely individual aspect of things, and is beginning to regard them as types or instances of a general law. Of course, the child is not fully conscious of any general principle. He does not separate the latter from its embodiment in the particular case, or put it into language even to himself. But, in order to infer, one must take the individual case as something more than a mere particular, as this which is only here and now. In the child's perception of resemblance between the present object and the one previously experienced, there is an implicit reference to a permanent type, or identity which persists through the two cases. In other words, when one asks what a perception of resemblance means, one sees that it implies an apprehension on the part of intelligence of something which is more than merely momentary.

The same quality or other element which is found in that object is also found in this. And the inference proceeds, that object was hot, therefore this object (having the same general nature, or being of the same type) is also hot. It is, of course, frequently impossible to formulate clearly the nature of the principle upon which we proceed, and, in cases like those cited, one may not be aware that it is present. But, I hope, it will now be clear that even in such instances the inference is based upon a permanent nature present in the two cases. We have already seen that where such an identical link is not present, it is impossible to pass, by means of inference, from a knowledge of one thing to another. As mere particulars, two phenomena occurring at different times are entirely isolated, and have nothing to do with each other. But as pieces of knowledge, facts which have been constituted by the interpreting function of Judgment, they are bound together by a common principle, the nature of a whole or system. This principle is, indeed, not anything *apart* from the facts connected, or in any way prior to them; but nevertheless something without which it would be impossible to understand their connection.

The conclusion of the matter, then, is that we never reason from one bare particular to another particular. More than that, we may say a fact which is merely particular — something which is only here and now — has no existence in knowledge. For knowledge lays hold of the universal aspect of things, their permanent significance. Intelligence sees the universal or typical nature in what is for sense a fleeting phenomenon. It

is only when the facts of sense are interpreted in this way, when their real nature is apprehended by thought, that they can be said to be known at all. Knowledge sees the universal in the particular, or reads the particular as a case of the universal. And when thus interpreted, the particular ceases to be a bare particular, and becomes an individual with a permanent nature of its own. When one reasons from an individual case, then, it is the universal or typical nature, not the particular or momentary existence, upon which the inference proceeds. If there were any merely particular facts in knowledge, we could never reason from them. But, as has been shown, the so-called particular facts, as elements of knowledge, possess a universal or typical aspect in virtue of which alone inference is possible.

§ 92. **Reasoning from Individual Cases to a Universal.** —There remains another question which is very closely related to the points already discussed in this chapter. We must admit that in inductive inference at least the starting-point is individual instances, though, as the last section showed, the latter, as used in reasoning, are something more than mere particulars. The problem which meets us, however, is this: How is it ever possible to get a universal conclusion from individual instances? It, of course, frequently happens that we cannot examine all the cases. What right then have we in these circumstances to state our conclusion generally — to assert, for example, that '*all* men are mortal,' or '*all* mosses cryptogams'?

It is often said that in such cases the general con-

clusion is never more than probable, and that its probability increases directly in proportion to the number of instances examined. Thus if A and B are conjoined only once in my experience, it is very improbable that the connection is a universal and essential one. But if they are found together ten times, the proposition, 'A is B' begins to have probability, which is, of course, greatly increased (without ever becoming more than probable however), if the conjunction is observed a hundred, or a thousand times. Now, there can be no doubt that the frequency of conjunction is, to a certain extent, a *practical test* of real or universal connection. Belief, as a *psychological* fact, is engendered by frequency of repetition. But the causes of our belief are here, as in many cases, quite different from the real or logical grounds. The fact that two phenomena have occurred together a hundred times, *in itself* affords no *logical ground* for affirming a universal connection between them, or that they will be connected the hundred and first time. Of course, as we have said, *psychological* belief or expectation would be engendered by the frequent conjunction; but the latter would supply no real or logical grounds. Practically, we are more certain to be right, if we generalize on the basis of a large number of observations, than if we proceed on the authority of a smaller number. But, as affording *logical* justification for our procedure, a hundred instances (if they are merely counted) are no better than one.

The truth is that a general conclusion does not depend for its logical justification upon the number of instances observed. Inference is not a matter of count-

ing instances at all, but is an intellectual insight into the nature of a general law or principle of connection. The problem of inductive inference is to discover this principle in the individual case, to penetrate beneath the surface, and read out of the individual phenomenon its real meaning or significance. To accomplish this usually requires an examination of many particular cases. We have more chances of learning the secret fully if we take as wide a survey as possible of the facts. A generalization based upon a small number of observations is pretty sure to be incorrect or inaccurate. But though of such great *practical* importance, the number of instances is *logically* indifferent. The essential point is to detect the general law or principle, and for this purpose one case may conceivably be as good as a hundred. Inductive inference, then, is not a process of passing from a certain number of cases to a general conclusion which always remains probable because it has no proper justification. But its real nature consists in the discovery, through the aid of examples, of a universal law of connection. We have already shown the part which the constructive imagination, guided by Analogy, plays in reaching this result (cf. §§ 60, 63).

It must be admitted that there are many cases where it is impossible to get beyond the fact that two phenomena are constantly conjoined in our experience. The grounds which should make this fact intelligible lie beyond our ken. Under circumstances of this kind, we are, of course, compelled to act on the presumption that the same order of events will continue to obtain. We may find that a certain medicine is followed by certain physiological consequences, without

being able to discover anything regarding the way in which the latter have been produced. And we may confidently predict that the same results will follow in a new case where the same medicine has been given. But it must be noticed that this is not the ideal of reasoning. Knowledge of the kind we have described is merely empirical, follows a rule of thumb without being able to give any account of itself. Moreover, even in such cases, it is always assumed that there is some general principle or law which may yet be discovered, and which is capable of explaining the facts known empirically.

References

J. S. Mill, *Logic*, Bk. II.

H. Spencer, *Principles of Psychology*, § 208.

W. James, *The Principles of Psychology*, Vol. II. Ch. XXVIII.

B. Bosanquet, *Logic*, Vol. II., pp. 176–179.

QUESTIONS AND EXERCISES

INTRODUCTION

CHAPTER I. — *The Standpoint and Problem of Logic*

1. What are some of the main characteristics of thought or thinking?

2. Explain the use of the verb *to think* in each of the following sentences: 'I do not know, but I think so;' 'If you think the matter over, you will come to the same conclusion.'

3. 'Words and phrases are often repeated without reflection, and their very familiarity is likely to prevent us from attempting to understand exactly what ideas they represent.' Give illustrations of this fact.

4. What do you mean by a science? How does 'scientific' knowledge differ from the knowledge of ordinary life?

5. What is the meaning of the word 'law' in the phrase 'a law of thought'? Compare the use of the word in such expressions as 'laws of nature,' 'the laws of the land.'

6. Is it true that Logic and Psychology have the same subject-matter?

7. Explain carefully how the problem of Logic differs from that of Psychology.

8. If we parallel Psychology with Morphology, and Logic with Physiology, what mental science will correspond to Embryology?

9. Illustrate by means of examples not used in the text the relation in which science and art, or theory and practice, stand to each other.

10. Criticise the following statement: 'Logic is not only a science; it is also an art, for it teaches us to reason correctly.'

11. What part does Introspection play in investigating logi cal questions?

12. In what sense may we say that the records of everything which the human race has accomplished form the material of Logic?

CHAPTER II. — *Historical Sketch*

1. 'The sciences have arisen in response to the practical needs of mankind.' Is this statement confirmed by the history of the origin and development of Logic?

2. 'Since each individual sees things from his own point of view, there is therefore nothing really true in itself, or good in itself.' Give some illustrations of the former part of this statement. What term would you use to describe the theory which the sentence expresses?

3. Explain what is meant by the statement that Socrates and Plato found a standard of truth and of conduct in the Concept.

4. Why was it not possible for Aristotle to lay down a complete theory of Inductive Reasoning?

5. What is Mill's theory regarding the relation of Induction and Deduction?

6. Describe the standpoint of Modern Logic.

PART I. — THE SYLLOGISMS

CHAPTER III. — *The Syllogism and its Parts*

1. Describe the general purpose and nature of the syllogism.

2. What is the principle upon which syllogistic reasoning depends? Why is it impossible to reason if this principle is violated?

3. Explain the distinction between the formal and real truth of an argument.

4. Arrange the following sentences as logical propositions, pointing out the logical Subject and the Predicate in each case: —

(*a*) Learning taketh away the wildness of men's minds.
(*b*) Dissipation wastes health.
(*c*) The exposition of a principle indirectly contributes to its proof.
(*d*) To me the meanest flower that lives can give thoughts that do often lie too deep for tears.
(*e*) The Alps consist of several parallel ranges.
(*f*) The travellers had found the city in ruins.

5. Point out the Premises and Conclusion in the following arguments, and supply any premise which may be wanting: —

(*a*) He is not indifferent to money; for he is a sensible man, and no sensible man despises money.
(*b*) All human productions are liable to error, and therefore all books, being human productions, are liable to error.
(*c*) All that glitters is not gold; for brass glitters.
(*d*) All bodies which move round the sun are planets; therefore the earth is a planet.
(*e*) Platinum is a metal, and therefore combines with oxygen.

6. How does Jevons describe Simple Apprehension? Is it possible to maintain that Apprehension, Judgment, and Reasoning are three distinct operations of mind?

Chapter IV. — *Terms*

1. Distinguish in the following list the terms which are usually (1) Singular, (2) General, and (3) Collective. If any

term may belong to more than one class, explain and illustrate its various uses: —

Niagara Falls,	an oak tree,	the United States Navy,
gold,	a dancing party,	Brooklyn Bridge,
chair,	the United States,	humanity,
a pack of cards,	city,	the centre of the earth.

2. Explain and illustrate the ambiguity in the use of the word 'all.'

3. In what two ways are the words Abstract and Concrete used? In what sense, if at all, can we say that Psychology and Logic are 'abstract' sciences?

4. Distinguish carefully between Contradictory and Opposite terms.

5. What are Correlative terms? Give at least three examples.

6. Mention the synonyms for Intension and Extension.

7. Explain the Extensional and Intensional use of the following terms: —

metal,	chair,	man,	Cæsar,	superstition,
justice,	student,	John Jones,	island,	emperor.

8. Criticise the statement that 'Extension and Intension stand in inverse ratio to each other.' What truth does it contain?

9. Invent a series of at least six terms which may be arranged so as gradually to increase in Extension.

10. What may be said in reply to Mill's contention that proper names are non-connotative?

CHAPTER V. — *Definition and Division*

1. Why is Definition necessary?

2. What is the distinction between extensive and intensive definition? What is a verbal definition?

3. In what two ways may we conceive the problem of Definition?

4. What do you understand by the Socratic Dialectic? Explain its purpose and mode of procedure.

5. Explain the terms: —

genus,	differentia,	infima species,
species,	summum genus,	sui generis.

6. Criticise the following definitions, pointing out what rules, if any, are violated by them: —

(1) Logic is the science of thought.
(2) A power is a force which tends to produce motion.
(3) Tin is a metal lighter than gold.
(4) A gentleman is a man who has no definite means of support.
(5) The body is the emblem or visible garment of the soul.
(6) Man is a vertebrate animal.
(7) Thunder-bolts are the winged messengers of the gods.
(8) A moral man is one who does not lie or steal or live intemperately.
(9) Cheese is a caseous preparation of milk.
(10) Evolution is to be defined as a continuous change from indefinite incoherent homogeneity to definite coherent heterogeneity of structure and function, through successive differentiations and integrations (Spencer).
(11) Oats is a grain which in England is generally given to horses, but in Scotland supports the people.

7. Give examples of terms which are indefinable, and explain why this is the case. What is the distinction between Description and logical Definition?

8. Define the following terms by giving the genus and differentia: —

science,	republic,	psychology,	island,
triangle,	monarchy,	gold standard,	import duty.

9. Examine the following Divisions and point out which are logical and which are not: —

(1) Living beings into moral and immoral.
(2) Men into saints and sinners.
(3) Religions into true and false.
(4) Man into civilized and black.
(5) Geometrical figures into rectilinear and non-rectilinear.
(6) Substances into material and spiritual.
(7) Metals into white, heavy, and precious.
(8) Elementary mental processes into sensations and affections.
(9) Students into those who are idle, those who are athletic, and those who are diligent.
(10) Books into scientific and non-scientific.

CHAPTER VI. — *Propositions*

1. What is a proposition? In what sense may a proposition be said to have parts?

2. Distinguish between Categorical and Conditional propositions.

3. What is meant by (*a*) the Quality, and (*b*) the Quantity, of propositions?

4. Arrange the following sentences in the form of logical propositions, and indicate the Quality and Quantity of each categorical proposition by the use of the letters A, E, I, and O: —

(1) Brevity has to be sought without sacrificing perspicuity.
(2) He that doeth these things is like to a man that buildeth his house upon a rock.
(3) Socrates declared knowledge to be virtue.
(4) Phosphorus does not dissolve in water.
(5) Nearly all the troops have left the town.
(6) Only ignorant persons hold such opinions.
(7) Few persons are proof against temptation.
(8) Over the mountains poured the barbarian horde.
(9) Except ye repent, ye shall all likewise perish.
(10) Neither gold nor silver is the proper standard of value.

5. How does formal logic interpret the relation between the subject and predicate of a categorical proposition? Does this view do full justice to the signification of propositions?

6. How would you represent by means of circles the proposition, 'gold is the most precious metal'?

7. What do you mean by the distribution of terms? Explain why negative propositions distribute the predicate, while affirmative propositions do not.

8. State precisely what is asserted by Proposition I. What forms may the diagrams which represent this proposition assume?

Chapter VII. — *The Interpretation of Propositions*

1. Why is it better to speak of the Interpretation of propositions than to use the term 'Immediate Inference'?

2. What is meant by the Opposition of propositions?

3. Explain the distinction between Contrary and Contradictory propositions.

4. If proposition O is false, what is known regarding the truth or falsity of A, E, and I?

5. What is the simplest proposition which must be established in order to disprove the following statements: (*a*) All men desire wealth. (*b*) No man is perfectly happy. (*c*) Some knowledge is not of any value. (*d*) Pain alone is evil. (*e*) All is not lost.

6. Give the contrary (or sub-contrary), and the contradictory of: (*a*) All metals are elements. (*b*) No coward need apply. (*c*) Socrates was the wisest man in Athens. (*d*) Not all men are brave. (*e*) No man but a traitor would have done this.

7. Give the Obverse of the following propositions: —

(1) All horses are quadrupeds.
(2) Good men are charitable.
(3) None of the captives escaped.
(4) Some of the planets are not larger than the earth.
(5) Some students do not fail in anything.
(6) All English dukes are members of the House of Lords.
(7) No illogical author is truly scientific.

8. Convert in at least one way: —

(1) All men are rational.
(2) Some metals are readily fusible.
(3) Perfect happiness is impossible.
(4) None of the captives escaped.
(5) Uneasy lies the head that wears a crown.
(6) Not every man could stand such hardships.
(7) None but the brave deserves the fair.
(8) Phosphorus will not dissolve in alcohol.
(9) Hydrogen is the lightest body known.
(10) The world is my idea.

9. Convert by contraposition: —

(1) All honest men are of this opinion.
(2) Oxygen can be prepared by heating potassium chlorate in a thin glass flask,

(3) Some of the enemy were not prepared to surrender.
(4) Not all who came to scoff remained to pray.
(5) A triangle is a plane figure bounded by three straight lines.
(6) The return of peace had given fresh confidence to the government party.

10. Describe the logical relation between each of the four following propositions: —

(1) All substances which are material possess gravity.
(2) No substances which possess gravity are immaterial.
(3) Some substances which are immaterial do not possess gravity.
(4) Some substances which do not possess gravity are immaterial. (Jevons.)

11. What is the Obverse of the Converse of, 'None of the planets shine by their own light'?

12. Can we logically conclude that because heat expands bodies, therefore cold contracts them? (Jevons.)

13. What is the logical relation, if any, between the two assertions in Proverbs xi. 1, 'A false balance is an abomination to the Lord; but a just weight is his delight'? (Jevons.)

Chapter VIII. — *The Syllogism and its Rules*

1. What is the relation of the Proposition and the Syllogism?

2. What is the function of the Middle Term in a Syllogism?

3. How are the major and minor terms, and the major and minor premises of a Syllogism distinguished?

4. Prove the seventh and eighth canon of the Syllogism, (*a*) by means of the previous rules, and (*b*) by the use of circles.

5. Construct an argument to illustrate the fallacy of ambiguous middle term.

6. Arrange the following arguments in the regular logical order of major premise, minor premise, and conclusion, and examine them to see whether they conform to the canons of the Syllogism: —

(1) Gold is not a compound substance; for it is a metal, and none of the metals are compounds.

(2) All national holidays are bank holidays, the bank will therefore be closed on the fourth of July.

(3) All cruel men are cowards, no college men are cruel, therefore no college men are cowards.

(4) Some useful metals are becoming rarer. Iron is a useful metal, and is therefore becoming rarer.

(5) This man shares his money with the poor, but no thief ever does this, therefore this man is not a thief.

(6) He who is content with what he has is truly rich. An envious man is not content with what he has; no envious man therefore is truly rich.

7. What does the Figure of an Argument depend upon? How do you distinguish the four figures?

CHAPTER IX. — *The Valid Moods and the Reduction of Figures*

1. Arrange the following arguments in logical order, and give the mood and figure in each case: —

(1) No P is M, Some S is M, Therefore some S is not P.	(2) All M is S, Some M is P, Therefore some S is P.

2. Name the premises from which valid conclusions may be drawn, no account being taken of figures: —

AA, EO, IA, IO, II, EE, EI, AE, EA, OO.

3. Prove the special canons of the fourth figure.

4. 'The middle term must be distributed once at least.' In what figures may it be distributed twice? What is the character of the conclusion when this occurs?

5. Prove generally that when the major term is predicate in its premise, the minor premise must be affirmative.

6. If the major term be distributed in its premise, but used undistributively in the conclusion, determine the mood and figure.

7. Explain why we can obtain only negative conclusions by means of the second figure and particular conclusions by means of the third figure.

8. What conclusions do AA, AE, and EA yield in the fourth figure? Explain.

9. Is it possible for both major and minor terms to be particular at the same time in the premises? If so, construct an argument where this is the case.

10. What do you understand by Reduction? Reduce the following argument to the first figure: —

> No fixed stars are planets,
> All planets are bright and shining,
> Therefore some bright and shining bodies are not fixed stars.

CHAPTER X. — *Abbreviated and Irregular Arguments*

1. Complete the following arguments, determine their mood and figure, and examine them to see if they violate any of the rules of the syllogism: —

(1) Blessed are the meek, for they shall inherit the earth.

(2) He must be a strong man, for he was on the crew.

(3) Zoöphytes have no flowers; therefore they are not plants.

(4) None but material bodies gravitate, therefore air is a material body.

(5) He has been a politician for years, and is therefore not to be trusted.

2. Illustrate the difference between the Progressive or Synthetic, and the Regressive or Analytic methods as employed in Mathematics and Psychology. May a science employ both methods at the same time?

3. Break up the concrete examples of Sorites given on pages 130, 131, into syllogisms.

4. Show generally why all the premises except the first in the Aristotelian Sorites must be universal.

5. Prove that in the Goclenian Sorites the first premise alone can be negative, and the last alone particular.

6. In the examples of arguments given on page 133, is there any middle term? If not, what serves as the standard of comparison?

CHAPTER XI.—*Hypothetical and Disjunctive Arguments*

1. What reasons are there for classifying the disjunctive proposition as conditional?

2. What are the rules of the hypothetical syllogism?

3. Is it ever possible to obtain a valid conclusion by denying the antecedent or affirming the consequent?

4. Determine which of the following hypothetical arguments are valid and which invalid; then express the latter in the categorical form, pointing out what are the categorical fallacies which result:—

(1) If a man is avaricious, he will be unhappy; but A is

unhappy, and we may therefore conclude that he is avaricious.

(2) If A is B, C is D, but A is B, therefore we may conclude that C is D.

(3) If the door were locked, the horse would not be stolen; but the horse is not stolen, therefore the door must have been locked.

(4) If man were not capable of progress, he would not differ from the brutes; but man does differ from the brutes, therefore he is capable of progress.

(5) If he had studied his lesson, he would have been able to recite; but he was able to recite, and therefore must have studied his lesson.

(6) If it becomes colder to-night, the pond will be frozen over; but it will not become colder to-night, therefore the pond will not be frozen over.

5. What aspects of thinking are emphasized by the categorical and hypothetical forms of reasoning respectively?

6. How far may the disjunctive proposition be regarded as an expression of ignorance, and what is the justification for the statement that it involves systematic knowledge?

7. To what fallacy is the disjunctive argument specially liable?

8. How would you criticise the dilemmatic arguments given on page 150?

CHAPTER XII. — *Fallacies of Deductive Reasoning*

1. What is the distinction between errors of interpretation and fallacies in reasoning?

2. Why is the detection of material fallacies a proper subject of logic?

3. If it is true that all the righteous people are happy, can

we conclude that all unhappy people are unrighteous? If so, how do we pass from the first statement to the second?

4. Can we proceed logically from the proposition, 'all good citizens vote at elections,' to 'all who vote in elections are good citizens'?

5. Does the statement that 'some sciences are useful,' justify the proposition that 'some useful things are not sciences'?

6. Mention the fallacies of Equivocation, and explain what is common to them all.

7. Explain the terms: *Petitio Principii*, *Circulus in probando*, *Argumentum ad hominem*, *Argumentum ad populum*.

8. Examine the following reasoning: 'The argument from design must be regarded as without value; for it has been rejected by Spinoza, Kant, Spencer, and Darwin.'

MISCELLANEOUS EXAMPLES

Arrange the following arguments whenever possible in regular logical order; determine whether or not they are valid; give the mood and figure of the valid categorical arguments; if any argument is invalid, point out and name the fallacy involved: —

1. All virtue is praiseworthy, and charity is a virtue, therefore charity is praiseworthy.

2. All colours are physical phenomena; but no sound is a a colour, therefore no sound is a physical phenomenon.

3. Some minerals are precious stones, all topazes are precious stones, therefore some minerals are topazes.

4. Some acts of homicide are laudable, therefore some cruel things are laudable.

5. If he has found the treasure, he is rich; but he has not found it, therefore he is not rich.

6. He must be a Democrat; for all the Democrats believe in Free Trade.

7. If only the ignorant despise knowledge, this man cannot be ignorant, for he praises it. (Edinburgh, 1892.)

8. Whatever is given on the evidence of sense may be taken as a fact; the existence of God, therefore, is not a fact, for it is not evident to sense. (St. Andrews, 1896.)

9. This explosion must have been occasioned by gunpowder; for nothing else would have possessed sufficient force.

10. This burglary is the work of a professional; for an amateur would not have been half so clever.

11. No stupid person can become President of the United States; therefore Mr. Cleveland and Mr. McKinley must both be men of ability.

12. Since almost all the organs of the body have some use, the vermiform appendix must be useful.

13. Every candid man acknowledges merit in a rival, every learned man does not do so; therefore learned men are not candid.

14. Every book is liable to error, every book is a human production, therefore all human productions are liable to error.

15. Learned men sometimes become mad; but as he is not learned, there is no danger of his sanity.

16. If this candidate used money to secure his election, he deserved defeat; but he did not use money in this way, and therefore did not deserve defeat.

17. All valid syllogisms have three terms; this syllogism is therefore valid, for it has three terms.

18. No persons destitute of imagination are true poets; some persons destitute of imagination are good reasoners; therefore some good reasoners are not true poets.

19. Only material bodies gravitate; ether does not gravitate.

20. In reply to the gentleman's arguments, I need only say that two years ago he advocated the very measure which he now opposes.

21. If he claims that he did not steal the goods, why, I ask, did he hide them as no thief ever fails to do?

22. If this therefore be absurd in fact and theory, it must also be absurd in idea, since nothing of which we can form a clear and distinct idea is impossible. (Hume, *Treatise of Human Nature.*)

23. Whatever is produced without a cause is produced by nothing, or in other words has nothing for its cause. But nothing can never be a cause. Hence every object has a real cause of its existence. (Hume, *Treatise.*)

24. Everything must have a cause; for if anything wanted a cause it would produce itself, that is, exist before it existed, which is impossible. (Hume, *Treatise.*)

25. If it be true, as Mr. Spencer thinks, that the past experience of the race has produced innate ideas and feelings, Weismann's denial of Use-inheritance would be refuted. Certainly, but it is just possible that Mr. Spencer's theory is not true.

26. Democracy is not a perfect form of government, for under it there are able men who do not get power; and so it allows men to get power who are not able.

27. Of university professors, some are zealous investigators, and some good teachers. A is an excellent teacher, and we may therefore conclude that he is not a zealous investigator.

28. Seeing that abundance of work is a sure sign of industrial prosperity, it follows that fire and hurricane benefit industry, because they undoubtedly create work. (St. Andrews, 1895.)

29. I will have no more doctors; I see that all of those who have died this winter have had doctors. (St. Andrews, 1896.)

30. If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease. (London, 1897.)

31. None but the wise are good, and none but the good are happy, therefore none but the wise are happy. (Edinburgh, 1897.)

32. Giving advice is useless. For either you advise a man what he means to do, in which case the advice is superfluous; or you advise him what he does not mean to do, and the advice is ineffectual. (London, 1897.)

33. No pauper has a vote, A B is not a pauper, therefore he has a vote. (St. Andrews, 1897.)

34. The love of nature is never found either in the stupid or the immoral man, therefore stupidity and virtue are incompatible. (Edinburgh, 1897.)

35. Not all educated persons spell correctly; for one often finds mistakes in the papers of University students.

36. Free Trade is a great boon to the workingman; for it increases trade, and this cheapens articles of ordinary consumption; this gives a greater purchasing power to money, which is equivalent to a rise in real wages, and any rise in real wages is a boon to the workingman.

37. If the train is late, I shall miss my appointment; if it is not late, I shall not reach the depot in time to go by it, therefore, in any case, I shall miss my appointment.

38. He who spareth the rod hateth his child; the parent who loves his child therefore spareth not the rod.

39. Whatever tends to withdraw the mind from pursuits of a low nature deserves to be promoted; classical learning does this, since it gives us a taste for intellectual enjoyments; therefore it deserves to be promoted.

40. As against the proposition that the formation of public libraries prevents private individuals from purchasing, and so decreases the sale of books, a writer urges that whatever encourages the reading of books encourages the buying of books. It is a library's purpose to encourage reading, and

hence the net result is rather to increase than to lessen purchases.

41. No reason however can be given why the general happiness is desirable, except that each person, so far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons. (Mill's *Utilitarianism.*)

42. This man is a Protestant; for he exercises the right of private judgment.

43. If the orbit of a comet is diminished, either the comet passes through a resisting medium, or the law of gravitation is partially suspended. But the second alternative is inadmissible. Hence if the orbit of a comet is diminished, there is present a resisting medium.

44. How do we know that our intuitive beliefs concerning the world are invariably true? Either it must be from experience establishing the harmony, or an intuitive belief must certify the correctness. Now experience cannot warrant such harmony except in so far as it has been perceived. Still more futile is it to make one instinctive belief the cause of another. Thus we cannot know that any intuitive belief is universally valid. (Bain.)

45. Which of the following are real inferences: (1) 'This weighs that down, therefore it is heavier'; (2) 'This piece of marble is larger than that, and therefore is heavier.'

46. The parts of pure space are immovable, which follows from their inseparability, motion being nothing but change of distance between any two things; but this cannot be between parts that are inseparable, which therefore must be at perpetual rest one amongst another.

47. If a body moves, it must move either in the place where it is, or in the place where it is not. But a body cannot move in the place where it is, nor yet in the place where it is not. Hence a body cannot move at all.

48. We have no perfect idea of anything but a perception. A substance is entirely different from a perception. We have therefore no idea of substance. (Hume.)

49. Every good government promotes the intelligence of the people, and no despotism does that. (Bain.)

50. He was too impulsive a man not to have committed many errors. (Bain.)

51. A true philosopher is independent of the caprices of fortune, for he places his chief happiness in moral and intellectual excellence.

52. Educated among savages, he could not be expected to know the customs of polite society. (Bain.)

53. No war is long popular; for every war increases taxation, and the popularity of anything that touches our pockets is very short lived.

54. The general object which all laws have, or ought to have, in common, is to augment the total happiness of the community; and therefore, in the first place, to exclude as far as may be everything that tends to subtract from that happiness: in other words, to exclude mischief. But all punishment is mischief; all punishment in itself is evil. Upon the principle of utility, if it ought at all to be admitted, it ought only to be admitted in as far as it promises to exclude some greater evil. (Bentham.)

55. Experiments for the purpose of ascertaining the functions of the various organs in animals cause pain, and as we are not warranted in causing pain to any sentient creature, such experiments are wrong.

56. Thou shalt not bear false witness against thy *neighbour*.

57. What is the use of all this teaching? Every day you hear of a fraud or forgery, by some one who might have led an innocent life, if he had never learned to read and write. (Edinburgh.)

58. Pious men only are fit to be ministers of religion; some men who have not received a college education are pious men, therefore such men are fitted to be ministers of religion.

59. What fallacy did Columbus commit when he proved that an egg could stand on end? (Jevons.)

60. No traitor is to be trusted, John is no traitor, and therefore is to be trusted.

61. Against what fallacy does the proverb, 'all that glitters is not gold,' warn us?

62. Livy describes prodigies in his history, therefore he is never to be believed.

63. The theory of evolution is true, for it is accepted by every scientific biologist.

64. The theory of evolution is not true, for it was not accepted by Agassiz, nor by Gladstone; moreover, you cannot accept this doctrine, for it is disclaimed by the authorities of your church.

65. The advantages which would accrue to the working-classes are not sufficient to justify Protection, neither are the advantages which it would bring to the farmers or the manufacturers, or to any other class in the community; Protection therefore has not enough advantages to justify it.

66. No man should be punished if he is innocent; this man should not be punished; therefore he is innocent.

67. He could not face bullets on the field of battle, and is therefore a coward.

68. We know that God exists because the Bible tells us so; and we know that whatever the Bible affirms must be true because it is of divine origin.

69. Nations are justified in revolting when badly governed, for every people has a right to good government. (Edinburgh.)

70. When Crœsus was about to make war upon Cyrus, King of Persia, he consulted the oracle at Delphi, and received for an answer that, if he should wage war against the Persians, he would overthrow a mighty empire.

71. England has a gold coinage, and is a very wealthy country, therefore it may be inferred that other countries having a gold coinage will be wealthy.

72. Your arguments against the philosophy of Hegel are of no value; for you uphold that of Schopenhauer, which is equally repugnant to common sense.

73. For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary; and it is ineffectual for the idle, and such as are indifferent to mental improvement; therefore the incitement of academical honours is either unnecessary or ineffectual.

74. Without order there is no living in public society, because the want thereof is the mother of confusion, whereupon division of necessity followeth; and out of division, destruction.

75. If it is always impossible not to sin, it is always unjust to punish. Now it is always impossible not to sin, for all that is predetermined is necessary, and all that is foreseen is predetermined, and every event is foreseen. Hence it is always unjust to punish. (Leibniz, *Theodicy.*)

76. If a gas is heated, its temperature rises; if its temperature rises, its elastic force increases; if its elastic force increases, the pressure on the walls of the containing vessel increases; therefore if a gas is heated, the pressure on the walls of the containing vessel increases. (Ray.)

77. The end of human life is either perfection or happiness; death is the end of human life, therefore death is either perfection or happiness.

78. If light consisted of material particles, it would possess momentum; it cannot consist of material particles, for it does not possess momentum.

79. This person is very learned, and very sociable, hence it follows that learning increases sociability.

80. Why advocate socialism? Until men become morally perfect, it is impossible; when they have become so, it will be unnecessary. (Edinburgh.)

81. The diameter of the earth is, in round numbers, forty millions of feet. Consequently the attraction of a sphere of the same mean density as the earth, but one foot in diameter, will be $\frac{1}{40000000}$ part the attraction of the earth; that is, $\frac{1}{40000000}$ of the weight of the body attracted. Consequently, if we should measure the attraction of such a sphere of lead, and find that it was just $\frac{1}{40000000}$ that of the weight of the body attracted, we would conclude that the mean density of the earth was equal to that of lead. But the attraction is actually found to be nearly twice as great as this; consequently a leaden sphere is nearly twice as dense as the average of the matter composing the earth. (Newcomb, *Popular Astronomy*.)

82. Mr. C. said that he was certain that the donors gave the property to the institution with a distinct and unanimous understanding as to its future use. The directors who acted for the institution in this transfer must necessarily have had an understanding, either the same as that of the donors, or different. If the understanding of the directors was the same as that of the donors, then they, the former, were unquestionably bound to live up to that understanding. If it was different, then the property was conveyed on a misunderstanding, and every dictate of honour and fair play would demand the return of the property.

PART II. — Inductive Methods

Chapter XIII. — *The Problem of Induction*

1. Explain why syllogistic logic is not a complete account of the nature of thinking.

2. In what sense is it possible to lay down the laws of scientific procedure?

3. In solving a complex scientific problem do we usually employ but a single method?

4. What can you say regarding the division of inductive methods into methods of Observation, and methods of Explanation?

5. Would it be permissible to add Experimental methods as a third and independent class?

6. What is the distinction between 'empirical' and 'scientific' knowledge?

7. What are the advantages to be derived from experiments in scientific work?

Chapter XIV. — *Enumeration and Statistics*

1. What is the justification for beginning our account of the inductive methods with Enumeration?

2. Explain what Jevons regards as 'Perfect' induction. Has this process any right to the name?

3. For what purpose are statistics employed? To what classes of phenomena are they applied?

4. What is meant by a phenomenon?

5. Explain how statistics may suggest causal laws, or confirm our expectation of them. May statistics also be used to disprove a proposed law of causal connection? Illustrate your answer.

6. Explain what is meant by the 'average,' and show how it is obtained.

7. How does the procedure of insurance companies differ from gambling?

Chapter XV. — *Causal Determination*

1. What are the two main principles upon which the canons proposed by Mill are founded?

2. Give the Canon of the Method of Agreement, and illustrate its use.

3. 'I have noticed that A always precedes B, it is therefore the cause of B.' Is this good reasoning?

4. What is meant by the 'Plurality of Causes'?

5. Under what disadvantages does the Method of Agreement labour? How is it supplemented?

6. State and illustrate the canon of the Method of Difference.

7. Why is this method applicable only to the spheres where experiment can be employed? Would it be safe to depend upon this method in determining the causes of social or political conditions?

Chapter XVI. — *Causal Determination* (*continued*)

1. Where do we employ the Joint Method?

2. What would it be necessary to establish in order to prove inductively that some change in the tariff laws was beneficial to the country?

3. 'One of the main characteristics of modern science is its *quantitative* nature.' Explain.

4. How does the law of Concomitant Variations assist us in determining causal relations?

5. In what two ways may the Method of Residues be applied?

6. Mention some discoveries to which the investigation of unexplained residues has led.

Chapter XVII. — *Analogy*

1. Why do we include Analogy among the methods of Explanation?

2. What do you mean by Analogy? What is the principle upon which it proceeds?

3. How is the word used in mathematical reasoning, and in physiology?

4. Into what Figure of the Syllogism does an argument from Analogy naturally fall? Is the argument formally valid, and if not, to what syllogistic fallacy does it correspond?

5. Explain how Analogy may suggest a true law or explanatory principle.

6. Why do we speak of Analogy as Incomplete Explanation?

7. If all analogical reasoning yields only probability, is not one analogy as good as another for purposes of inference? If not, upon what does the *value* of an inference from Analogy depend?

Chapter XVIII. — *The Use of Hypotheses*

1. How do you distinguish the terms 'theory' and 'hypothesis'?

2. What is an hypothesis, and how is it used?

3. Do hypotheses play any part in assisting Observation? Explain and illustrate.

4. Give some instances in which hypotheses have proved injurious, and have misled people regarding the nature of facts.

5. 'Hypotheses are formed by the imagination working in dependence upon facts and guided by analogy.' Explain.

6. What are the steps in the proof of an hypothesis?

7. Explain what part is played by Induction and Deduction respectively in using hypotheses.

8. What canons have been laid down to which a good hypothesis must conform? Why are the first and third of these rules of little value?

9. Explain why an unverifiable hypothesis is not worth discussing.

CHAPTER XIX. — *Fallacies of Induction*

1. What is the source of fallacy? How far is it true that the study of Logic can protect us from fallacies?

2. How do you classify Inductive Fallacies?

3. Explain and illustrate the following fallacies: *Question-begging Epithet, Equivocation, Fallacies due to Figurative Language.*

4. Explain and illustrate the tendency of the mind to neglect negative cases.

5. Is it an easy matter to 'tell just what we saw and heard' at a particular time?

6. What do you mean by *post hoc ergo propter hoc?* Why may we take this as the general type of inductive fallacies?

7. What did Bacon mean by the Idols of the Cave?

8. 'Every age, as well as every individual, has its idols.' Explain this statement.

MISCELLANEOUS EXAMPLES

Analyze the examples of inductive reasoning given below, and point out what methods are employed, indicating also whether or not the conclusion is completely established: —

1. In my experience A has been invariably preceded by B, and we may therefore conclude that it is the cause of it.

2. Scarlet poppies, scarlet verbenas, the scarlet hawthorn, and honeysuckle are all odourless, therefore we may conclude that all scarlet flowers are destitute of odour.

3. What inference, if any, can be drawn from the following statement: 'In nine counties, in which the population is from 100 to 150 per square mile, the births are 296 to 100 marriages; in sixteen counties, with a population of 150 to 200 per square mile, the births are 308 to 100 marriages'?

4. The great famine in Ireland began in 1845 and reached its climax in 1848. During this time agrarian crime increased very rapidly, until, in 1848, it was more than three times as great as in 1845. After this time it decreased with the return of better crops, until, in 1851, it was only 50 per cent more than it was in 1845. It is evident from this that a close relation of cause and effect exists between famine and agrarian crime. (Hyslop.)

5. Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyl of carbon-dioxide gas under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions of the leaves of an illuminated plant were covered with black paper, no starch was found in these portions.

6. Jupiter gives out more light than it receives from the sun. What is the obvious conclusion, and by what method is it reached?

7. What methods would you employ in order to test the truth of the proposition, *omne vivum ex vivo?*

8. War is a blessing, not an evil. Show me a nation that has ever become great without blood-letting.

9. If wages depend upon the ratio between the amount of labor-seeking employment, and the amount of capital devoted to its employment, the relative scarcity or abundance of one factor must mean the relative abundance or scarcity of the other. Thus capital must be relatively abundant where wages are high, and relatively scarce where wages are low. Now, as the capital used in paying wages must largely consist of the capital-seeking investment, the current rate of interest must be the measure of its relative abundance or scarcity. So if it be true that wages depend upon the ratio between the amount of labor-seeking employment, and the capital devoted to its employment, then high wages must be accompanied by low interest, and, reversely, low wages must be accompanied by high interest. This is not the fact but the contrary. (George.)

10. Construct an inductive argument to prove that some article of food, or some habit, is beneficial or injurious to you, and analyze your reasoning, showing the methods which you have employed.

11. Some comets have been observed to have the same orbits as certain meteoric showers. The hypothesis is suggested that all meteoric showers may represent the débris of disintegrated comets. Biela's comet having been missing for some time, it was accordingly predicted that when next due it would be replaced by a meteoric shower. This prediction was verified by observation.

12. Tyndall found that of twenty-seven sterilized flasks containing infusion of organic matter, and opened in pure Alpine air, not one showed putrefaction; while of twenty-three similar flasks, opened in a hay-loft, only two remained free from putrefaction after three days. He concluded that putrefaction is due to floating particles in the air.

13. 'Whether or not a bad theory is better than none, depends upon circumstances.' Examine this statement, and point out what are some of the circumstances of which mention is made.

14. It is said that a general resemblance of the hills near Ballarat in Australia to the Californian hills where gold had been found suggested the idea of digging for gold at Ballarat. (Minto.)

15. There are no great nations of antiquity but have fallen to the hand of time; and England must join them to complete the analogy of the ages. Like them she has grown from a birth-time of weakness and tutelage to a day of manhood and supremacy; but she has to face her setting. Everything that grows must also decay. (Edinburgh, 1893.)

16. Goldscheider proved that muscular sensations play no considerable part in our consciousness of the movements of our limbs, by having his arm suspended in a frame and moved by an attendant. Under these circumstances, where no work devolved on his muscles, he found that he could distinguish as small an angular movement of the arm as when he moved and supported it himself.

17. Goldscheider also proved that the chief source of movement-consciousness is pressure sensations from the inner surface of the joints, by having his arm held so that the joint surfaces were pressed more closely together, and finding that a smaller movement was now perceptible.

18. Wages in the United States are higher than in England, because the former country is a republic and has a protective tariff.

19. It does not follow that an institution is good because a country has prospered under it, nor bad because a country in which it exists is not prosperous. It does not even follow that institutions to be found in all prosperous countries, and not

to be found in backward countries, are therefore beneficial. For this at various times might confidently have been asserted of slavery, of polygamy, of aristocracy, of established churches; and it may still be asserted of public debts, of private property in land, of pauperism, and of the existence of distinctly vicious or criminal classes. (George.)

20. Explain the procedure of the *reductio ad absurdum* form of argument.

21. It may be a coincidence merely; but, if so, it is remarkably strange that while the chloroform has not changed, while the constitutions of the patients have not changed, where the use of the inhaler is the rule there are frequent deaths from chloroform; whilst in Scotland and Ireland, where the use of the inhaler is the exception, deaths are proportionally rare.

22. We should think it a sin and shame if a great steamer, dashing across the ocean, were not brought to a stop at a signal of distress from the mere smack. . . . And yet a miner is entombed alive, a painter falls from a scaffold, a brakeman is crushed in coupling cars, a merchant fails, falls ill and dies, and organized society leaves widow and child to bitter want or degrading alms. (George, *Protection and Free Trade.*)

23. Manufacturing countries are always rich countries; countries that produce raw material are always poor. Therefore, if we would be rich, we must have manufactures, and in order to get them, we must encourage them. . . . But I could make as good an argument to the little town of Jamaica . . . in support of a subsidy to a theatre, I could say to them: all cities have theatres, and the more theatres it has the larger the city. Look at New York! . . . Philadelphia ranks next to New York in the number and size of its theatres, and therefore comes next to New York in wealth and population. . . . I might then drop into statistics . . . and point to the fact that when theatrical representations began in this country, its popu-

lation did not amount to a million, that it was totally destitute of railroads, and without a single mile of telegraph wire. Such has been our progress since theatres were introduced that the census of 1880 showed we had 50,155,783 people, 90,907 miles of railroad, and $291,212\frac{9}{10}$ miles of telegraph wires. (George, *Protection and Free Trade.*)

24. What methods would you employ to investigate the connection between changes in the barometer and in the weather?

25. In Sir Humphry Davy's experiments upon the decomposition of water by galvanism, it was found that, besides the two components of water, oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. The insight of Davy conjectured that there might be some hidden cause of this portion of the effect: the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. . . . By the substitution of gold vessels for glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the cause, whatever it was, was still in operation. . . . He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further until no more than slight traces of them were perceptible. An experiment determined this: the machine was put under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and the alkali. (Gore, *The Art of Scientific Discovery.*)

26. Properties known to exist in potassium have been predicted of and found to exist in rubidium; for instance, the carbonates of sodium and potassium are not decomposed by a red heat, neither are those of rubidium, or cæsium. Some of the statements which are true of chlorine have been found to be true, in varying degrees, of bromine and iodine. . . . After I had found the molecular change in antimony electro-deposited from its chloride, I sought for and discovered it in that deposited from its bromide and iodide; and after having found magnetic changes in iron by heat, I also found similar ones in nickel. (Gore, *The Art of Scientific Discovery*.)

27. What inductive fallacy may David be said to have committed when he said in his haste that all men are liars?

28. It has been found that linnets when shut up and educated with singing larks — the skylark, woodlark, or titlark — will adhere entirely to the songs of these larks, instead of the natural song of the linnets. We may infer, therefore, that birds learn to sing by imitation, and that their songs are no more innate than language is in man. (Hyslop.)

29. We observe very frequently that very poor handwriting characterizes the manuscripts of able men, while the best handwriting is as frequent with those who do little mental work when compared with those whose penmanship is poor. We may, therefore, infer that poor penmanship is caused by the influence of severe mental labor. (Hyslop.)

30. Galileo describes his invention of the telescope as follows: This then was my reasoning; this instrument [of which he had heard a rumor] must either consist of one glass, or of more than one; it cannot be of one alone, because its figure must be either concave or convex, or comprised within two parallel superficies, but neither of these shapes alter in the least the objects seen, although increasing or diminishing them; for it is true that the concave glass diminishes, and that the

convex glass increases them; but both show them very indistinctly, and hence one glass is not sufficient to produce the effect. Passing on to two glasses, and knowing that the glass of parallel superficies has no effect at all, I concluded that the desired result could not possibly follow by adding this one to the other two. I therefore restricted my experiments to combinations of the other two glasses; and I saw how this brought me to the result I desired. (Quoted by Gore, *The Art of Scientific Discovery*.)

31. Darwin was struck by the number of insects caught by the leaves of the common sun-dew. It soon became evident to him that "Drosera was excellently adapted for the special purpose of catching insects." . . . As soon as he began to work on Drosera, and was led to believe that the leaves absorbed nutritious matter from the insects, he began to reason by analogy from the well-understood digestive capacity of animals. . . . Having by analogy established the power of digestion in plants, analogy led him to seek in plants the elements that do the work of digestion in animals. Bringing together what was known of plants, he pointed out that the juices of many plants contain an acid, and so one element of a digestive fluid was at hand; and that all plants possess the power of dissolving albuminous or proteid substances, protoplasm, chlorophyl, etc., and that "this must be effected by a solvent, probably consisting of ferment together with an acid." After writing the last-quoted sentence, he learned that a ferment which converted albuminous substances into true peptones had been extracted from the seeds of the vetch. (Cramer, *The Method of Darwin*.)

32. Strongly impressed with the belief that some 'harmonic' relation must exist among the distances of the several planets from the sun, and also among the times of their revolution, Kepler passed a large part of his early life in working out a

series of guesses at this relation, some of which now strike us as not merely most improbable, but positively ridiculous. His single-minded devotion to truth, however, led him to abandon each of these hypotheses in turn so soon as he perceived its fallacy by submitting it to the test of its conformity to observed facts. . . . But he was at last rewarded by the discovery of that relation between the times and the distances of the planetary revolutions, which with the discovery of the ellipticity of the orbits, and of the passage of the radius vector *over equal areas in equal times* has given him immortality as an astronomical discoverer. But . . . he was so far from divining the true *rationale* of the planetary revolutions that he was led to the discovery of the elliptical orbit of Mars by a series of happy accidents . . . whilst his discovery of the true relations of times and distances was the *fortunate* guess which closed a long series of *un*fortunate ones, many of which were no less ingenious.

Now it was by a grand effort of Newton's *constructive* imagination, based on his wonderful mastery of geometrical reasoning, that, starting with the conception of two *forces*, one of them tending to produce continuous uniform motion in a straight line, the other tending to produce a uniformly accelerated motion towards a fixed point, he was able to show that if these *dynamical* assumptions were granted, Kepler's laws, being consequences of them, must be universally true. And it was his still greater glory to divine the profound truth that the fall of the moon towards the earth — that is the deflection of her path from a tangential line to an ellipse — *is a phenomenon of the same order* as the fall of a stone to the ground. (Gore, *The Art of Scientific Discovery*.)

33. After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of thunder and lightning were the same as those produced by

electricity. Lightning travels in a zig-zag line, and so does an electric spark; electricity sets things on fire, so does lightning; electricity melts metals, so does lightning. Animals can be killed by both, and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another? (Buckley, *A Short History of Natural Science.*)

34. How did Franklin proceed to verify the hypothesis stated in the last example?

35. Galileo discovered by means of his telescope that Jupiter has four moons, instead of one like the earth, and he regarded this discovery as a confirmation of the Copernican theory. Explain the nature of the reasoning involved in reaching this conclusion.

36. That the period of tide should be accidentally the same as that of the culmination of the moon, that the period of the highest tide should be accidentally the same as the syzygies, is possible *in abstracto;* but it is in the highest degree improbable: the far more probable assumption is, either that the sun and moon produce the tide, or that their motion is due to the same grounds as the motion of the tide. (Hibben.)

37. During the retreat of the Ten Thousand a cutting north wind blew in the faces of the soldiers, sacrifices were offered to Boreas, and the severity of the wind immediately ceased, which seemed a proof of the god's causation. (Hibben.)

38. A nectary implies nectar, but Sprengel had come to the conclusion that *orchis morio* and *orchis maculata,* though furnished with nectaries, did not secrete nectar. Darwin examined the flowers of *orchis morio* for twenty-three consecutive days, looking at them after hot sunshine, after rain, and at all hours;

he kept the spikes in water and examined them at midnight and early the next morning. He irritated the nectaries with bristles, and exposed them to irritating vapors. He examined flowers whose pollinia had been removed, and others which would probably have them soon removed. But the nectary was invariably dry.

He was thoroughly convinced, however, that these orchids require the visits of insects for fertilization, and that insects visit flowers for the attractions offered in the way of nectar, and yet that in these orchids the ordinary attraction was absent. In examining the orchids he was surprised at the degree to which the inner and outer membranes forming the tube or spur were separated from each other, also at the delicate nature of the inner membrane, and the quantity of fluid contained between the two membranes. He then examined other forms that do secrete nectar in the ordinary way, and found the membranes closely united, instead of separated by a space. "I was therefore led to conclude," he says, "that insects penetrate the lax membrane of the nectaries of the above-named orchids and suck the copious fluid between the two membranes." He afterwards learned that at the Cape of Good Hope moths and butterflies penetrate peaches and plums, and in Queensland a moth penetrates the rind of the orange. These facts merely proved his anticipation less anomalous than it had seemed. (Cramer, *The Method of Darwin.*)

39. Construct an hypothesis to explain some fact of your experience, and explain how it may be either verified or overthrown.

40. When Darwin began to work on *Drosera* he was led to believe that the leaves absorbed nutritious matter from insects. He then reasoned by analogy from the well-understood digestive capacity of animals. He made preliminary 'crucial' experiments by immersing some leaves of *Drosera*

in nitrogeneous and others in non-nitrogeneous fluids of the same density to determine whether the former affected the leaves differently from the latter. This he found to be the case. He then experimented with solid animal matter and found that the leaves are capable of true digestion. Analogy then led him to seek in plants the elements that do the work of digestion in animals. He pointed out that the juices of many plants contain an acid, and so one element of a digestive fluid was at hand; and that all plants possess the power of dissolving albuminous or proteid substance-protoplasm, chlorophyl, and that this must be effected by a solvent consisting probably of a ferment together with an acid. Afterwards he learned that a ferment which converted albuminous substances into true peptones had been extracted from the seeds of the vetch. (Cramer, *The Method of Darwin*, pp. 95–99.)

41. In opposition to the facts stated above, Tischutkin maintains that the 'digestion' of insectivorous plants is not accomplished in the same way as in animals, but is due to bacteria: that the pepsin is not a secretion of the plant, but a by-product of the activity of the bacteria. Suppose that this theory is true, and Darwin's false, what would you say regarding the character of the latter's reasoning?

PART III. — The Nature of Thought

Chapter XX. — *Judgment the Elementary Process*

1. What objections are there to speaking of thought as 'a thing like other things'?

2. What is the general law of Evolution? Explain what is meant by a change from the homogeneous to the heterogeneous.

3. What general conclusions are reached by the application of the law of Evolution to the thought-process?

4. What do you understand by Judgment? How does a simple judgment differ from sensation?

5. In what sense may our judgments be said to be the union of two concepts?

6. Would the doctrine that in knowing we first have Simple Apprehension, then as separate intellectual processes, Judgment and finally Inference, agree with the general evolutionary view of consciousness? Explain fully.

Chapter XXI. — *The Characteristics of Judgment*

1. What do you understand by the universality of judgments? What is the distinction between the universality of a judgment and that of a proposition?

2. How would you prove that all judgments are universal?

3. Is any judgment necessary in itself? If not, whence do judgments derive their necessity?

4. What is the argument by which it has been maintained that there must be judgments or principles which are in themselves necessary?

5. Explain how it is possible for a judgment to be at once both analytic and synthetic.

6. Explain what is meant by a 'system' of knowledge.

7. When judgment brings new facts into relation to what we already know, does the old body of knowledge undergo any modification?

Chapter XXII. — *The Laws of Thought*

1. In what sense can we speak of a law of Thought?

2. Explain what is meant by the law of Identity.

3. How has this law been interpreted by Boole and Jevons?

4. What does Jevons mean by the 'substitution of similars,' and how does he propose to employ this principle?

5. What objections are there to employing the sign of equality to represent the relation between the subject and predicate of a judgment?

6. Explain how the law of Identity is related to the characteristics of judgment treated in the last chapter.

7. What is the meaning of the law of Contradiction?

8. Explain the use of the law of Excluded Middle.

CHAPTER XXIII. — *Types of Judgment*

1. Why do we begin with judgments of Quality?

2. Explain how we pass in the development of intelligence from Quality to Quantity.

3. In what sense is it true that judgments of Quantity never give us the real nature of things, but only their relation to something else?

4. What is meant by anthropomorphic causes? How are they distinguished from scientific causes?

5. What new element did the discovery of the law of the Conservation of Energy introduce in the causal conception as employed in certain sciences?

6. Why cannot this new extension have any application in the field of the mental sciences?

7. How does the standpoint of judgments of Individuality differ from that of judgments of Causality?

CHAPTER XXIV. — *Inference*

1. How does Inference differ from Judgment? In what sense may it be said that it is an extension of the latter process?

2. Does the passage from Judgment to Inference illustrate the general law of Logical Evolution? Explain.

3. In the development of our knowledge, which usually comes first, premises or conclusion?

4. How is it possible to pass from the known to the unknown?

5. Explain under what circumstances only an Inference is possible.

6. What is the common element in both Induction and Deduction? How do they differ?

CHAPTER XXV. — *Rational and Empirical Theories*

1. Who are the great historical representatives respectively of Rationalism and Empiricism?

2. Explain the method and procedure of Rationalism.

3. What is the great instrument of knowledge according to Rationalism? What according to Empiricism?

4. State as clearly as you can the various points at issue between the two schools.

5. Explain Mill's theory that we always reason from one particular fact to another. How far do you agree with his conclusions?

6. Is it true that we obtain a general law by summing up particulars?

7. Is there any *direct* and necessary connection between the number of instances and the induction of the general law?

8. Criticise Jevon's theory of 'Perfect Induction' as stated on page 187.

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